Correlations between shear wave velocity and geotechnical parameters in Norwegian clays

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ABSTRACT
The purpose of this paper is to present guidelines and correlations to assists geotechnical engineers in estimating V_s profiles in Norwegian clays in the absence of site-specific data. For this, a database of in situ V_s measurements and standard geotechnical engineering material properties for Norwegian clays has been established. The database allowed the development of several empirical correlations between in situ V_s and basic soil properties, cone penetration parameters, undrained shear strength and 1D compression parameters. Based on the results from regression analyses, empirical functions based on cone penetrometer data to determine the best estimate in situ V_s of Norwegian clay are recommended to use when in situ measurements of V_s at the site are not available. Relationships based on undrained shear strength can also be used in practice as presented herein.

Keywords: *In situ* shear wave velocity, clay, undrained strength, compression parameters.

1 INTRODUCTION

The small-strain shear modulus of soils, G_{max}, is an important parameter for many geotechnical design applications, including site characterization, settlement analyses, seismic hazard analyses, site response analysis and soil-structure interaction. This is typically associated with strains on the order of 10^{-3}% or less. According to elastic theory, G_{max} may be calculated from the shear wave velocity (V_s) using the following equation:

$$G_{max} = \rho \cdot V_s^2 \qquad (1)$$

Where G_{max} is the shear modulus (in Pa), V_s is the shear wave velocity (in m/s), and ρ is the density (in kg/m^3).

G_{max} and V_s are primarily functions of soil density, void ratio, and effective stress, with secondary influences including soil type, age, depositional environment, cementation and stress history Hardin and Drnevich (1972).

G_{max} can be measured in the laboratory using a resonant column device or bender elements. As suggested by Kramer (1996), while the void ratio and stress conditions can be recreated in a reconstituted specimen, other factors such as soil fabric and cementation cannot. Laboratory testing requires very high-quality, undisturbed samples which is often a challenging and expensive task in soft and sensitive clays. Additionally, laboratory tests only measure G_{max} at discrete sample locations, which may not be representative of the entire soil profile.

Unlike laboratory testing, in situ geophysical tests do not require undisturbed sampling, maintain *in situ* stresses during testing, and measure the response of a large volume of soil. *In situ* measurement of V_s has become the preferred method for estimating the small strain shear properties and has been incorporated into site classifications systems and ground motion prediction equations worldwide.

In the absence of site-specific measurement, guidelines for estimating V_s profiles based on correlations with *in situ* penetration tests, soil index parameters and undrained shear strength may be used, recognizing that these indirect methods introduce greater uncertainties. The main objective of this paper is to present such guidelines for estimation of V_s in Norwegian clays.

2 DATA AND METHODS

The data used for correlation purposes originates from a total of 29 sites (Fig. 1). Out of these sites, 15 are located in south-eastern Norway while 13 are in mid Norway. The last site included in the database is the Bothkennar clay site in Scotland where much work has been carried out over the last 30 years (including testing of block samples by NGI), see for example Long et al. (2008). The reader is referred to NGI (2015) for a detailed overview of all sites in the database.

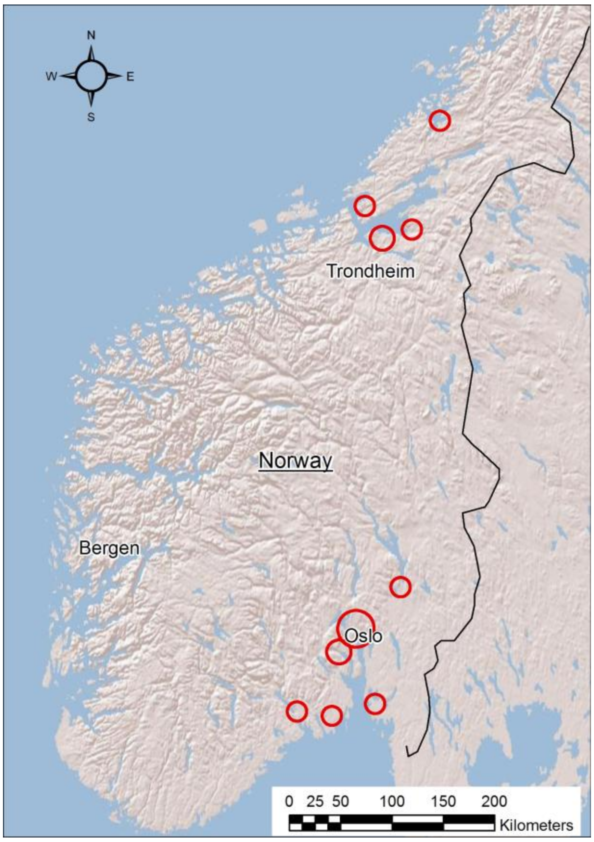

Figure 1 Overview map showing location of study sites included in database.

2.1 Measurement of in situ V_s

In situ V_s measurement was carried out at several Norwegian clay sites during the last decades for research purposes and/or as a part of construction projects. Source of existing data includes e.g. papers by Long and Donohue (2007) and Long and Donohue (2010), L'Heureux et al. (2013). In the present study previously published information was assemble with new field data. The *in situ* V_s data was acquired at most of the sites using the non-invasive method called multichannel analysis of surface wave (MASW). In addition comparative *in situ* V_s data was collected using the seismic cone penetrometer (SCPTU; 7 sites), cross-hole test (CHT; 5 sites) and spectral analysis of surface wave (SASW; 4 sites).

2.2 Available soil properties

The compiled database contains index and engineering properties obtained from classification tests, strength tests and consolidation tests. The database includes index properties such as total unit weight, water content, clay content, remoulded shear strength, sensitivity and Atterberg limits. Also, engineering properties such as undrained shear strength (s_u), net cone resistance (q_{net}), *in situ* effective vertical stress (σ'_{v0}) and 1D compression parameters were available to this study. Only data from high quality samples is used in this study (c.f. Lunne et al. 1998).

The Norwegian clays in the database are of marine or glacimarine origin. Natural water content (w) data range between 20 and 80% with most of the data in the range between 40 to 50% (Fig. 2). The plastic index (I_p) being defined as the difference between the liquid and plastic limits is presented in Fig. 3. Most of the plasticity index data vary between 5 and 20%. The clay content of the soil tested ranges from 10 to 70% with most of the data in the range between 30 to 50% (Fig. 4). The effective vertical stress in the database varies between 10 and 240 kPa with the highest number of observations at around

100 kPa corresponding to a depth of approximately 6-8 m below ground surface.

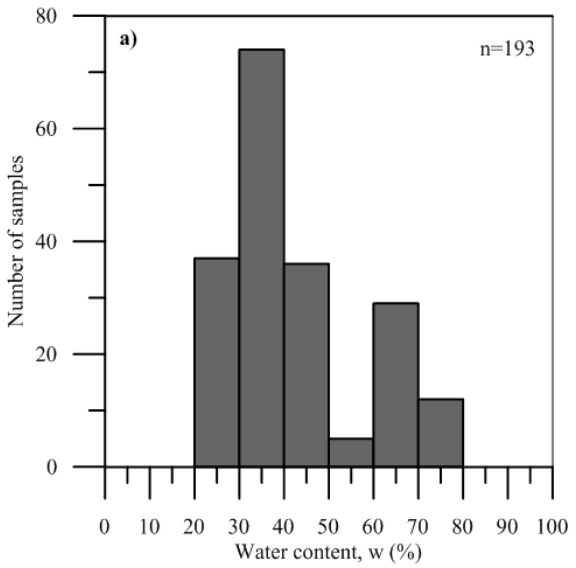

Figure 2 Range of water content in the database.

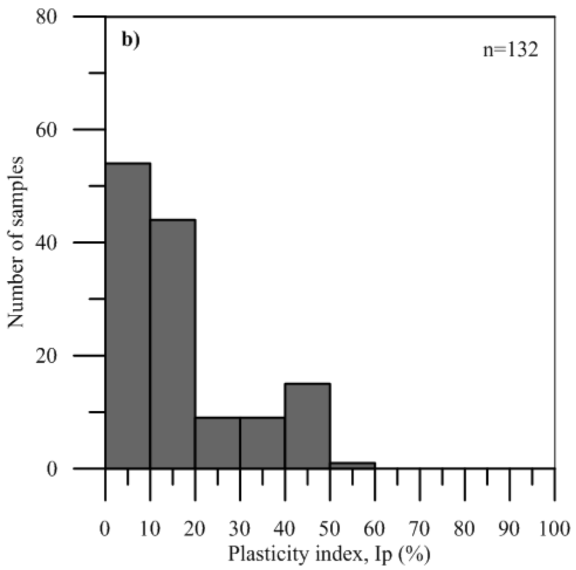

Figure 3 Range of I_p in the database.

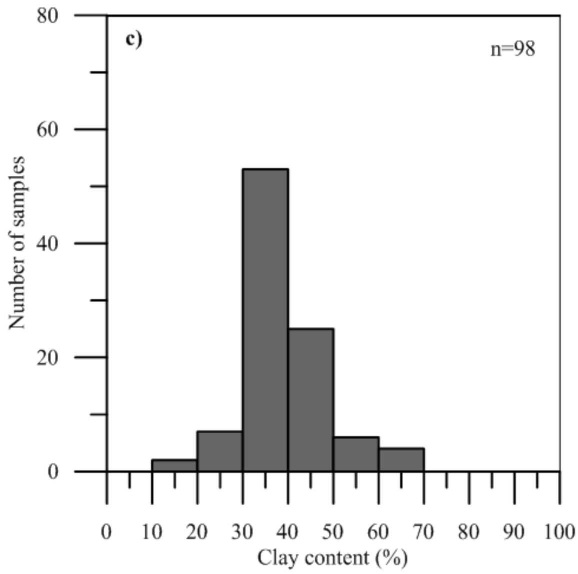

Figure 4 Range of clay content in the database.

Most of the clays in the database have developed some apparent overconsolidation due to aging. The overconsolidation ratio (OCR) data range between 1.0 and 8 with most of the OCR data falling between 1.5 and 2.0, indicating that most of the soil samples in the database are normally consolidated to lightly over-consolidated. Hence, the developed correlations below may not be valid for moderately to heavily overconsolidated clays.

The *in situ* shear wave velocity (V_s) data range between 50 and 300 m/s with the majority of the data between 120 and 250 m/s (Fig. 5). With the exception of Onsøy and Farriseidet the data follows a very similar depth pattern. V_s values are typically 120 m/s at ground level and increase to 180 m/s and 200 m/s at 10 m depth and 12 m depth respectively. The very soft high water content organic clays at Onsøy and especially Farriseidet show much lower values of V_s.

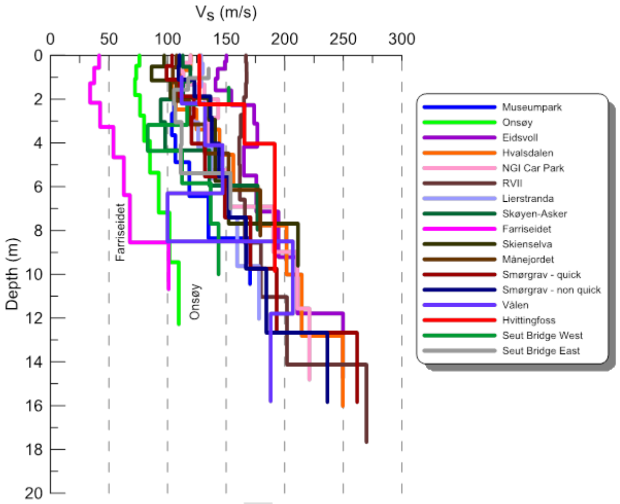

Figure 5 In situ shear wave velocity profile for sites in Southern Norway.

Shear wave velocity data for the Trondheim area in mid Norway are shown on Fig. 6. Broadly speaking the data can be divided into two groups. The main group show similar values to those from Southern Norway with V_s increasing from about 100 m/s at ground level to 200 m/s at about 12 m depth. There is a second group of sites all located in south and south-west Trondheim (comprising the Rosten, Saupstad, Okstad and Hoseith sites) with higher values. Here the V_s values reach 300 m/s at 10 m to 12 m. The very soft clay at Dragvoll show the lowest values on average. Generally it can be seen that all the Trondheim data fits with those from South Norway except for the Rosten (high OCR at the bottom of slope), Hoseith and Okstad sites.

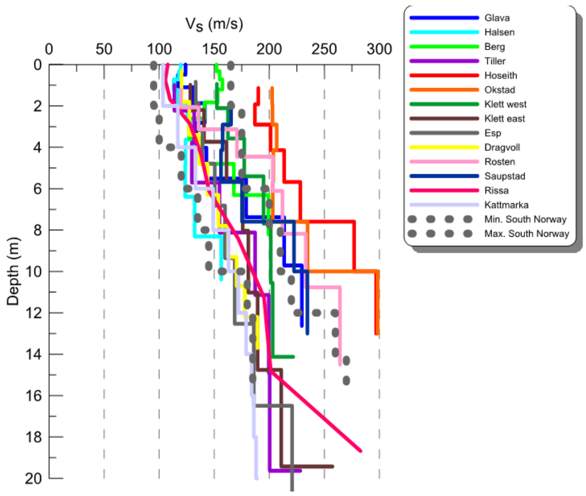

Figure 6 In situ shear wave velocity profile for sites in the Trondheim region.

3 CORRELATION WITH INDEX PROPERTIES

Correlations between index parameters and V_s or G_{max} can provide rapid estimates useful for preliminary design and for verifying *in situ* and laboratory results. Hardin (1978) suggested that G_{max} for clays depends on the *in situ* (or applied) stress (σ'), void ratio (e), and OCR. It has been shown, however, that the effects of OCR are, to a large extent, taken into account by the effect of void ratio and could be neglected, see for example (Leroueil and Hight, 2003). The empirical equation describing the influence of the controlling factors on G_{max} can then be written as follows:

$$G_{max} = SF(e)(\sigma_v^{'}\sigma_h^{'})^n p_a^{(1-2n)} \tag{2}$$

Where *S* is a dimensionless parameter characterizing the considered soil; *F*(*e*) is a void ratio function; σ'_v and σ'_h are the vertical and horizontal effective stresses, respectively; *n* is a parameter indicating the influence of stress; and p_a is the atmospheric pressure.

Figure 7 presents the relationship between V_s and σ'_{v0} for all sites in the database. Results show a clear tendency for V_s to increase with σ'_{v0}. The best fit equation for the data gives a regression coefficient of 0.68.

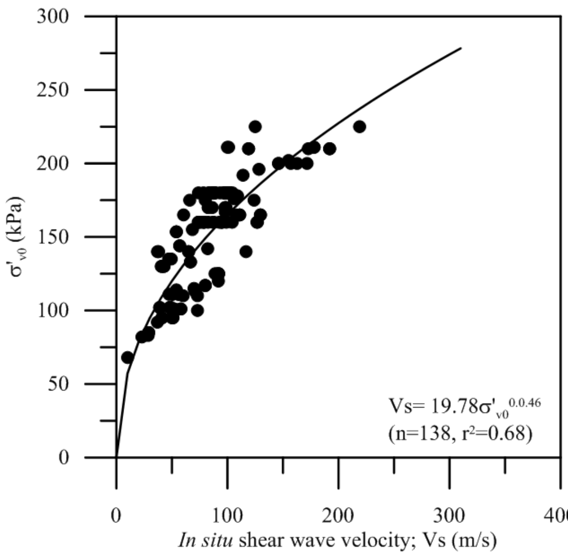

Figure 7 In situ V_s against vertical effective stress for all sites in the database.

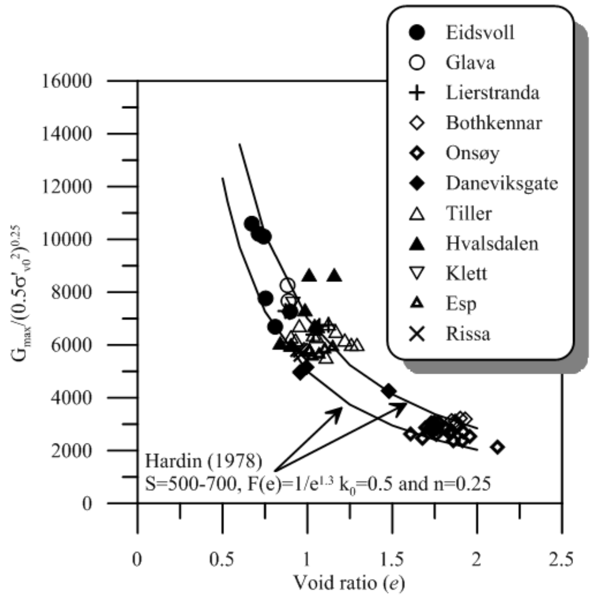

Figure 8 Relationship between G_{max} normalized according to Hardin (1978) and Hight and Leroueil (2003) and e.

The void ratio in the database was calculated using:

$$e_0 = \frac{G_s\,\gamma_{w\,(1+w)}}{\gamma_{tot}} - 1 \qquad (3)$$

Where G_s is the specific gravity of soil solids, γ_w is the unit weight of water, w the water content, and γ_{tot} the total unit weight of the soil. G_{max} values were normalized by the corresponding *in situ* vertical effective stress (σ'_{v0}). G_{max}/σ'_{v0} typically varies between 250 and 1000 in the database.

In Figure 8 the data have been normalized using Eq. [2]. Two lines have been added corresponding to S = 500-700, $F(e) = 1/e^{1.3}$, K_0' = 0.5 (where K_0' is the coefficient of earth pressure at rest), and n = 0.25. It can be seen that the fit is good and that S ranges from 500 to 700. This further confirms that G_{max} values for Norwegian clays are consistent with those from a large volume of other published experimental data.

For other correlations between index properties (e.g. Ip or w) and *in situ* Vs data from the Norwegian clay database, the reader is refered to NGI (2015) and L'Heureux and Long (submitted).

Table 1 Example of available CPTU-Vs correlations for clays.

Study/Reference	Number of data pairs	r^2	V_s (m/s) or G_{max} (kPa)
(Tanaka et al., 1994)			$G_{max} = 50 \cdot (q_t - \sigma_{v0})$
(Hegazy and Mayne, 1995)	406	0.890	$V_s = 14.13 \cdot (q_c)^{0.359} \cdot (e_0)^{-0.473}$
(Hegazy and Mayne, 1995)	229	0.780	$V_s = 3.18 \cdot (q_c)^{0.549} \cdot (f_s)^{0.025}$
(Mayne and Rix, 1995)	339	0.830	$V_s = 9.44 \cdot (q_c)^{0.435} \cdot (e_0)^{-0.532}$
(Mayne and Rix, 1995)	481	0.740	$V_s = 1.75 \cdot (q_c)^{0.627}$
(Piratheepan, 2002)	20	0.910	$V_s = 11.9 \cdot (q_c)^{0.269} \cdot (f_s)^{0.108} \cdot D^{0.127}$
(Mayne, 2006)	161	0.820	$V_s = 118.8 \cdot \log(f_s) + 18.5$
(Long and Donohue, 2010)	35	0.613	$V_s = 2.944 \cdot (q_t)^{0.613}$
(Long and Donohue, 2010)	35	0.758	$V_s = 65 \cdot (q_t)^{0.15} \cdot (e_0)^{-0.714}$
(Long and Donohue, 2010)	-	0.777	$V_s = 1.961 \cdot (q_t)^{0.579} \cdot (1 + B_q)^{1.202}$
(Taboada et al., 2013)	274	0.94	$V_s = 14.4 \cdot (q_{net})^{0.265} \cdot (\sigma'_{v0})^{0.137}$
(Taboada et al., 2013)	274	0.948	$V_s = 16.3 \cdot (q_{net})^{0.209} \cdot \left(\frac{\sigma'_{v0}}{w}\right)^{0.165}$

4 CORRELATION WITH CONE PENETRATION DATA

The piezocone penetration test (CPTU) is a common tool used for characterization of soft and sensitive clay deposits. Several studies have explored relationships between *in situ* V_s and parameters such as CPTU tip resistance (q_c), corrected tip resistance (q_t), cone net resistance (q_{net}), sleeve friction (f_s), pore pressure parameter (B_q), normalized cone resistance (Q_t), effective stress (σ'_v) and void ratio (e).

An overview of some of the most popular V_s prediction equations found in the literature for clays is presented in Table 1. For consistency, some of the equations have been modified to use of SI units: q_c, q_t, q_{net}, f_s and (σ'_v) are in kPa. The number of points used to develop each correlation equation is presented as well as the coefficient of determination (r^2).

Following the relationships proposed by Taboada et al. (2013), multiple regression analyses were conducted on the Norwegian clay database to provide power function expressions for *in situ* V_s in terms of q_{net}. The relationship with the highest coefficient of correlation using q_{net}, and σ'_{v0} is:

$$V_s = 8.35 \cdot (q_{net})^{0.22} \cdot (\sigma'_{v0})^{0.357} \qquad (4)$$

The coefficient of determination r^2 is 0.73 and a total of 115 datasets were used in the regression analysis. The trend between the *in situ* measured V_s and the prediction given by Eq. [4] is illustrated in Figure 9. The figure shows that most of the predicted values of V_s are within 20 % of the measured V_s.

The prediction given by equation [4] was improved when the water content was introduced giving rise to the following expression:

$$V_s = 71.7 \cdot (q_{net})^{0.09} \cdot \left(\frac{\sigma'_{v0}}{w}\right)^{0.33} \qquad (5)$$

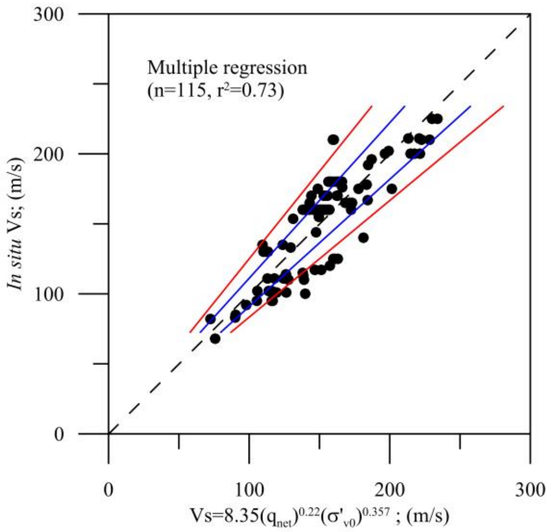

Figure 9 Comparison of measured and predicted V_s as a function of q_{net} and σ'_{v0}. Blue lines show +/-10% envelope while red lines show +/- 20%.

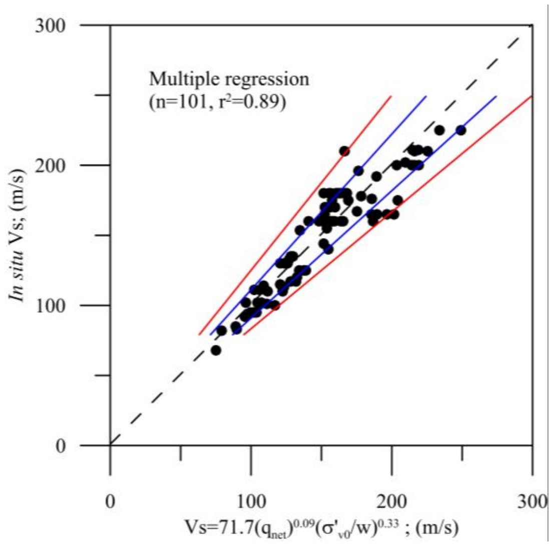

Figure 10 Comparison of measured and predicted V_s as a function of q_{net} , σ'_{v0} and w. Blue lines show +/-10% envelope while red lines show +/- 20%.

The coefficient of determination r^2 is 0.89 and a total of 101 datasets were used in the analyses. The trend between *in situ* measured V_s and the expression given in Eq. [5] is presented in Fig. 10. When using Eq. [5] most of the predicted values of V_s are within 10-15 % of the measured V_s. Equations 4 and 5 are similar to those presented by Taboada et al. (2013) for clays from the Gulf of Mexico (see Table 1). However, the empirical factors vary greatly. *In situ* V_s for Norwegian clays seem to be more strongly controlled by water content and vertical effective stresses, and to a lesser extent by the net cone resistance.

5 CORRELATION WITH UNDRAINED SHEAR STRENGTH

Similar to CPTU penetration-based correlations, relationships between V_s and undrained shear strength (s_u) for clays can be developed since both properties depend on common parameters.

The undrained shear strength values obtained from direct simple shear tests (DSS) on Norwegian clay samples are plotted against *in situ* shear wave velocity in Fig. 11. The results show an increase in $s_{u,DSS}$ with increasing V_s. The best fit is given by Eq. 6 with a regression coefficient (r^2) of 0.91.

$$V_s = 14.87\, s_{u,DSS}^{0.69} \tag{6}$$

Equation 6 can also be used to assess undrained shear strength from V_s measurements by rewriting the relationship and solving for s_u as follow:

$$s_{u,DSS} = 0.02\, V_s^{1.45} \tag{7}$$

The data in Fig. 11 is compared to the relationships proposed by Andersen (2004) (i.e. $G_{max}/s_{u,DSS}$= 800 – 900). Note that to compare with the relationships proposed by Andersen (2004) we made use of Eq. [1] by varying the density between 1.6 and 1.9 Mg/m^3 and the empirical factor between 800 and 900. Figure 11 shows the 2 extreme lines from the Andersen (2004) relationship. The fit is good at low Vs value, but large difference arise for higher Vs results. The reason for these differences may come from the fact that the relationships proposed by Andersen (2004) are based on laboratory measurements of V_s and G_{max}, whereas *in situ* V_s data are used in this study.

Correlations between *in situ* V_s data and undrained shear strength from CAUC and CAUE triaxial tests have also been established based on data collected in this study. For more details the reader is referred to NGI (2015) and L'Heureux and Long (submitted).

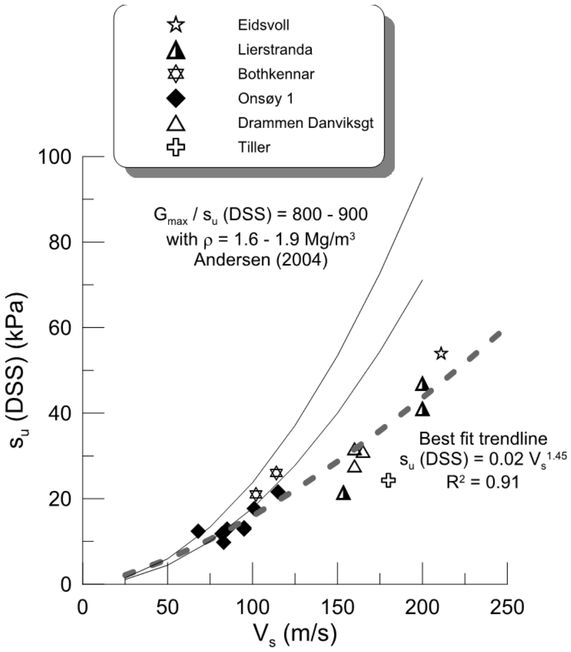

Figure 11 Results of in situ shear wave velocity against undrained shear strength from direct simple shear tests (s_{uDSS}).

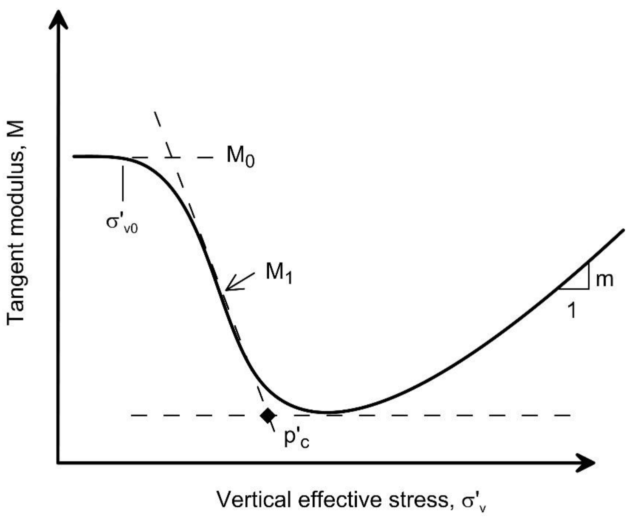

Figure 12 NGIs interpretation of the classical Janbu tangent modulus versus stress model.

6 CORRELATION WITH 1D COMPRESSION PARAMETERS

In this section the *in situ* V_s measurements in the database are compared Janbu's classical 1D compression parameters presented in Figure 12.

The relationship between M_0 and M_1 and V_s is shown on Figure 13 and 14, respectively.

Reasonable correlations would be expected here as V_s is a function of the current state of stress. Both M_0 and M_1 increase with increasing V_s as expected. The scatter in the data increases for increasing V_s and the greatest variation is for the highly overconsolidated Eidsvoll and Hvalsdalen clay. The best fit power trend lines shown give a reasonable r^2 values for both M_0 and M_1.

Values of the preconsolidation stress (p_c' as determined by the method presented in Figure 12) are plotted against V_s on Fig. 15. A good correlation is expected here as the shear wave velocity is strongly dependent on the maximum past stress experienced by the clay. The relationship between p_c' and V_s is good and the best fit power function has an r^2 value of 0.81 (Fig. 15).

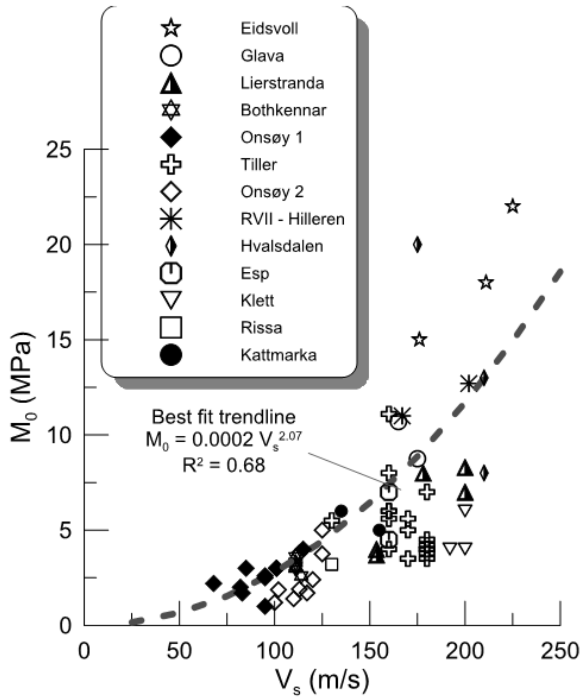

Figure 13 Results of Mo versus in situ V_s.

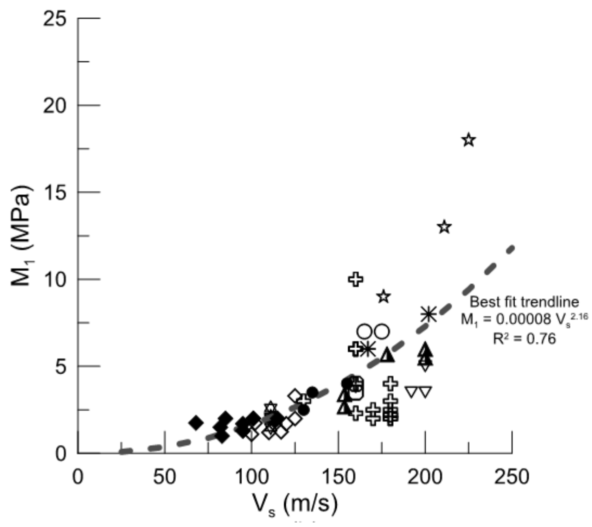

Figure 14 Results of M_1 versus in situ V_s.

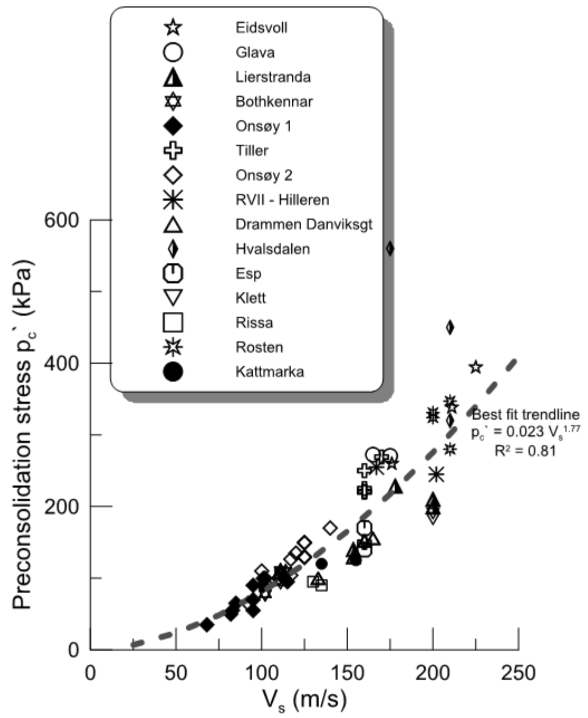

Figure 15 Results of p'_c versus in situ V_s.

The variation in the modulus number *m* versus shear wave velocity is shown on Fig. 16. There is a clear tendency for an increase in *m* with increasing V_s. However the fit is not as good for M_0, M_1 and p_c'. This is not surprising as you would expect V_s to represent the current state of stress not at some arbitrary higher stress stiffness.

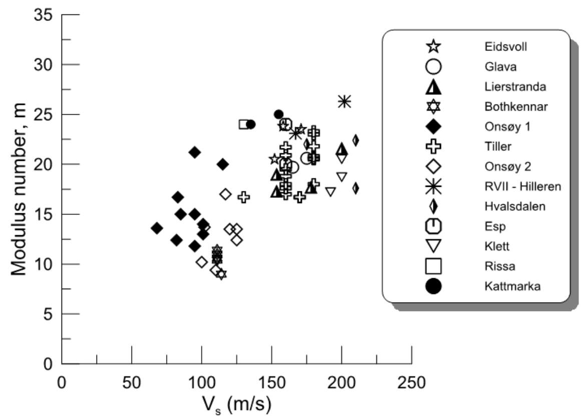

Figure 16 Modulus number versus V_s.

7 CONCLUSIONS AND RECOMMENDATIONS

The purpose of this study is to present guidelines and correlations to assist geotechnical engineers in estimating V_s profiles in Norwegian clays in the absence of site-specific data. For this, a database of *in situ* V_s measurements and standard geotechnical engineering material properties for Norwegian clays has been established. The database allowed the development of several empirical correlations between *in situ* V_s and basic soil properties, cone penetration parameters, undrained shear strength and 1D compression parameters. Based on the results from regression analyses, we recommend the use of empirical functions based on cone penetrometer data to determine the best estimate *in situ* V_s of Norwegian clay when *in situ* measurements of V_s at the site are not available. Relationships based on undrained shear strength can also be used in practice. Note that the relationships presented herein can be used either to evaluate V_s from a given soil property, or the way around to evaluate soil properties from V_s.

In general, it is recommended that engineers consider all available data including available relationships, *in situ* measured V_s profiles, and site-specific geotechnical data. The use of correlations in geotechnical engineering should be limited to the conditions for which they were developed and calibrated. The recommendations presented in this report should be used in conjunction with the engineer's own experience and engineering judgment. Site-specific correlations may be developed based on a limited number of site-specific V_s measurements and using a similar functional form.

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