The influence of grain size distribution and grain shape on the small strain shear modulus of North Sea Sand

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ABSTRACT
Small strain shear modulus of three North Sea sand samples and one onshore sand sample from the North Sea coast has been measured and the results are compared with existing empirical equations. Appreciable differences between the measured data and predictions are presented as functions of sand type, void ratio and stress level. The discrepancies are possibly influenced by amount of rounded particles in the sand composition.

Keywords: small strain shear modulus, particle shape, North Sea, sand composition.

1 INTRODUCTION

More than fifty years ago (Hardin & Richart, 1963) proposed an empirical relationship allowing estimation of the small strain shear modulus G_{max} in [kPa] from void ratio:

$$G_{max} = A\frac{(a-e)^2}{1+e}p^n \quad (1)$$

where e is the void ratio [decimal], p is the mean confining pressure in [kPa], $A, a,$ and n are material constants.
The relationship was developed based on experiments with Ottawa and crushed quartz sands. Originally two sets of material constants for round ($A = 6900, a = 2.17, n = 0.5$) and angular ($A = 3200, a = 2.97, n = 0.5$) grain shape were established.
The Hardin equation therefore takes the grain shape into account and predicts an increase of the shear modulus with the mean effective stress and a decrease with the void ratio. Equation (1) has been widely used as a first estimate for the G_{max}-values for different sands.
Further modifications also include a dimension-true form by Hardin & Drnevich (1978) and a dimensionless form by Wichtmann & Triantafyllidis (2014).
Wichtmann & Triantafillydis (2009) proposed the following empirical relations to calculate the material constants $A, a,$ and n for each sand independently as a function of coefficient of uniformity $C_u = d_{60}/d_{10}$:

$$A = 1563 + 3.13\, {C_u}^{2.98} \quad (2)$$

$$a = 1.94 \cdot exp\,(-0.066C_u) \quad (3)$$

$$n = 0.40\, {C_u}^{0.18} \quad (4)$$

These coefficients do not converge to Hardin's coefficients neither for angular nor for round grain shape. The relationships were elaborated on the basis of artificial grain size distribution (GSD) curves, which are linear in semilogarithmic scale and verified them also on gap-graded, stepwise linear and smoothly shaped GSD curves (Wichtmann & Triantafyllidis, 2014). They found that these extended relationships predict development of G_{max} reasonably well. In this study both the original Hardin & Richart (1963) and the improved Wichtmann & Triantafyllidis (2009) relationships are applied in the form of Equation (1) on three sands from the North Sea and one onshore sand with natural grain size distributions. It was found that both empirical relationships show appreciable deviations from the measured data. This study is going to show:

- whether this deviation is statistically significant
- what kind of parameters may influence this deviation.

2 MATERIAL

2.1 Study area

In the Quaternary the southeastern North Sea and northern Germany was affected by a series of glacial, terrestrial and marine controlled depositional or erosional processes, which resulted in a complex stratigraphy of repeated glacial, fluvial and lacustrine or marine sedimentation, interrupted by phases of erosion. Meltwater discharge beneath ice margins generated several overdeepened tunnel valleys, which are filled by a typical pattern of coarse-grained meltwater sediments at the base and the flanks overlain by fine-grained, often stratified, glaciomarine and/or glaciolacustrine sediments (Lutz et al. 2009). The sand samples S1-2 were recovered along the flanks of an Elsterian tunnel valley (Hepp et al. 2012) at the depth of 43.5 and 16.4 m below seafloor (mbsf), whereas sand sample S3 was obtained from the valley surroundings (5.5 mbsf). The onshore sand sample S4 was taken from a proximal fluvial deposit near a Saalian terminal moraine complex Altenwalde, Germany (Sindowski, 1965).

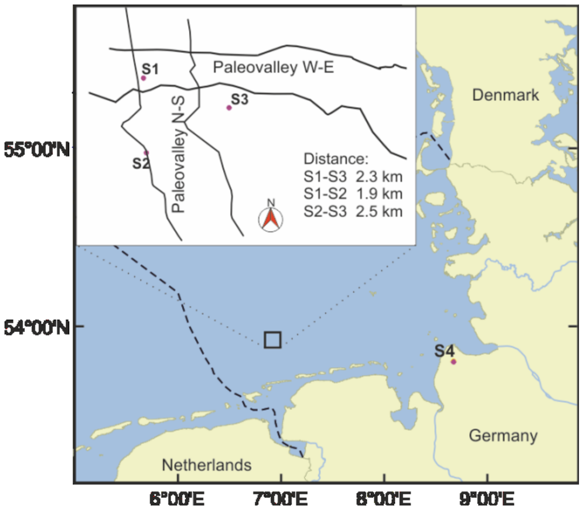

Figure 1. Study area with locations of sands S1, S2, S3 and S4 in relation to glacial tunnel valleys and the distance between them.

2.2 Classification parameters for sands tested

The sand samples are composed of almost 100% quartz, S1 and S2 reveal some cementation between its grains. Grain size distributions for each sample are shown in Figure 2.

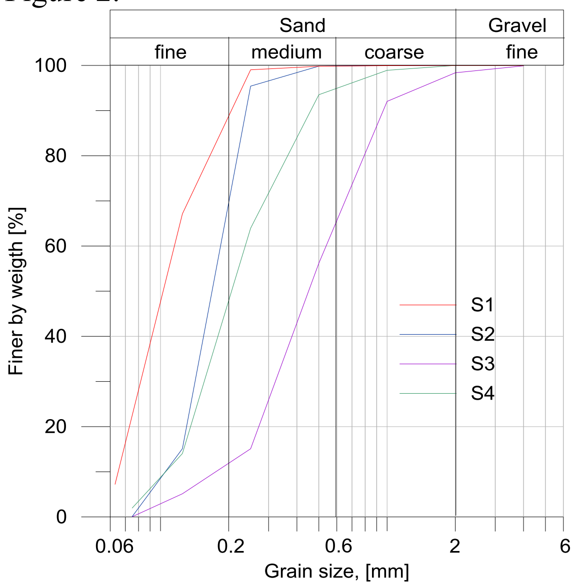

Figure 2. Grain size distributions

Mean grain size (d_{50}), coefficient of uniformity (C_u), minimum and maximum void ratios (e_{min}, e_{max}) according to NGI in-house procedure e.g. Blaker et al. (2015) as well as grain density (ρ_{grain}) measured in a helium pycnometer are shown in Table 1. For further testing the samples were separated at 2 mm by dry sieving.

Table 1. Index properties

Sand	d_{50} [mm]	C_u [/]	e_{min} [/]	e_{max} [/]	ρ_{grain} [g/cm³]
S1	0.123	2.15	0.62	0.98	2.67
S2	0.177	1.92	0.60	0.90	2.64
S3	0.453	3.55	0.39	0.69	2.67
S4	0.215	2.2	0.53	0.79	2.644

The grain size range of the sieves used is conform to the grain size classes in φ ($\varphi = -\log_2 d[mm]$). The normality of the grain size distribution (e.g. χ^2-test) and its moments calculated in the φ-space are shown in Table 2.

Table 2. Moments and the χ^2 -probability of the grain size distribution in φ units.

Sand	μ	σ	μ_3	μ_4	$F(\chi^2)$
S1	3,16	0,648	0,106	4,06	0,838
S2	2,51	0,414	0,130	6,61	0,836
S3	1,08	0,969	0,128	3,58	0,937
S4	2,12	0,835	-0,250	3,89	0,878

With μ mean, σ standard deviation, μ_3 skewness, μ_4 kurtosis of the grain size distribution. $F(\chi^2)$ is the cumulative distribution function of the χ^2 statistic.
The grain roundness was estimated visually using roundness charts after Powers (1953) as weighted average over 100 grains retained in each sieve with the mesh size w_i.

$$shape = \sum_{i<I} shape_i \cdot w_i \qquad (5)$$

Where $shape_i$ is the percentage of specific grain shape retained in the sieve with the mesh size i, w_i is the percentage of grains retained by weight in the sieve with the mesh size i. I is the maximal mesh size used for investigation
The grain shapes are shown in Table 3.

Table 3. Grain shape properties (A- Angular, S/A- Subangular, S/R- Subrounded, R-Rounded).

Sand	*I* mm	A [/]	S/A [/]	S/R [/]	R [/]
S1	0.25	0.327	0.277	0.288	0.108
S2	0,5	0.21	0.41	0.32	0.06
S3	1	0.29	0.36	0.21	0.14
S4	0,5	0.01	0.44	0.34	0.21

3 SMALL STRAIN STIFFNESS

The small strain stiffness was determined during 16 isotropically consolidated drained (CID) triaxial tests instrumented with bender elements at Norwegian Geotechnical Institute. Specimens were prepared in four relative densities at approximately D_r=50%, 65%, 80% and 95%. The specimens were built in in six layers using undercompaction technique (Ladd 1978) with a low water content. Initial water contents after saturation are shown in the Table 3.

Table 4. Initial water contents (%) after sample saturation

Sand	D_r=95 %	D_r=80 %	D_r=65 %	D_r=50 %
S1	24,30	26,32	28,13	30,44
S2	23,25	25,43	27,09	28,57
S3	20,77	22,64	23,83	25,26
S4	15,31	17,20	18,82	20,32

Shear waves were triggered and received using piezoceramic bender elements. Details concerning bender element technique are described in Dyvik & Madshus (1985).
v_s measurements were performed during consolidation at effective isotropic confining pressures of 100, 200 and 400 kPa. At each loading stage the specimen was first subjected to a consolidation period of at least 120 min, then v_s was measured before the next stress increment was applied.

3.1 Repeatability of G_{max}

In order to estimate the influence of the internal fabric a repeatability test was carried out. For this purpose three specimens of sand S4 were prepared with D_r=95%.The shear wave velocities were measured at three pressure stages for each specimen as described above. The results are shown in Figure 3 and Table 5.
All three specimens show an increase of the G_{max} with the mean effective stress. The gradient $\frac{\partial G_{max}}{\partial p}$ between two consequtive pressure stages is almost the same for all three specimens. The values of the test 1B agree well with the average values. The measured values deviate from the mean between 14.5%

at $p = 100\ kPa$ and 9.7% at $p = 400\ kPa$ (Table 5). The deviation appears to decrease with the stress level.

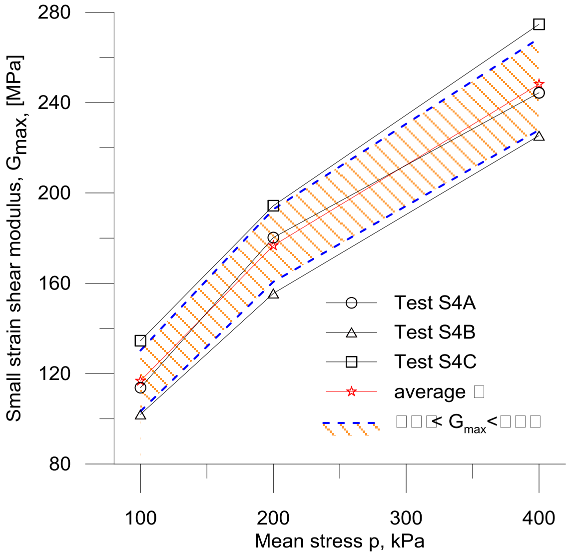

Figure 3. Value range of the repeatability test on sand S4 specimens prepared with Dr=95%. Red stars present the average values over three tests for each stress level, dashed area depicts the range of one standard deviation.

Table 5. Average (μ, [MPa]), standard deviation (σ, [MPa]) and deviation of the results from the average for S4 prepared with D_r=95% at different stress levels p[kPa].

P	μ	σ	$(G_{max}^{meas} - \mu)/G_{max}^{meas}$		
			S4A	S4B	S4C
100	117	13,47	-0,026	-0,145	0,132
200	177	16,04	0,020	-0,136	0,091
400	248	20,3	-0,015	-0,101	0,097

3.2 G_{max} for different sands

G_{max} for four investigated sands are combined with the empirical predictions in Figure 4. As expected G_{max} decreases with void ratio and increases with rising confining pressure. The bulk of the data generally follows the Hardin empirical equation but reveal considerable scatter at comparable void ratios. The Wichtmann equations show a steeper decrease of the G_{max} with the void ratio and seems to fit the laboratory data better for p=400 kPa.

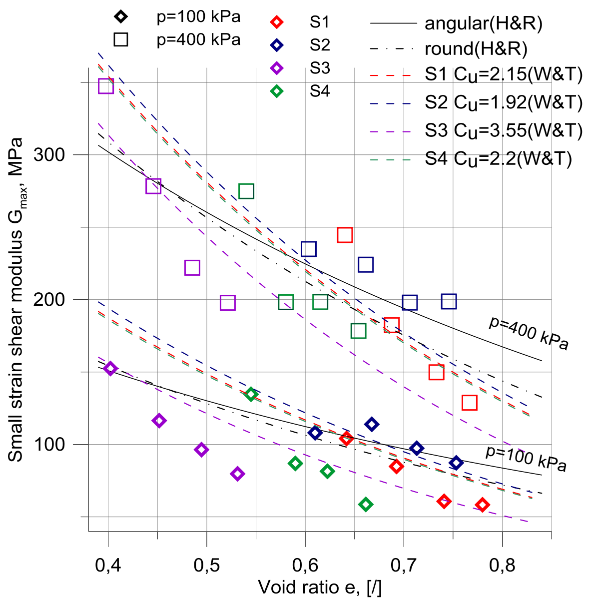

Figure 4. Test results for two stress levels. Black solid and dot-dashed lines depict empirical equations after Hardin& Richart (1963) (Marked as H&R). Coloured dashed lines depict empirical equations after Wichtmann & Triantafyllidis (2009) (Marked as W&T) for each sand with corresponding colour. The C_u-values for each sand used to calculate empirical constants after Wichtmann & Triantafyllidis (2009) are shown in brackets. In order to maintain the readability of the figure the test results and empirical predictions for the stress stages p=100 kPa and p=400 kPa are shown. The stress stage p=200 kPa is omitted.

3.3 Residual analysis

The relative residuals r_i between measured and predicted small strain shear moduli G_{max} were determined by:

$$r_i = \frac{G_{max\ i}^{predicted} - G_{max\ i}^{measured}}{G_{max\ i}^{measured}} \tag{6}$$

and the root mean square error RMSE as measure of the prediction quality was calculated by:

$$RMSE = \sqrt{\sum_i r_i^2} \tag{7}.$$

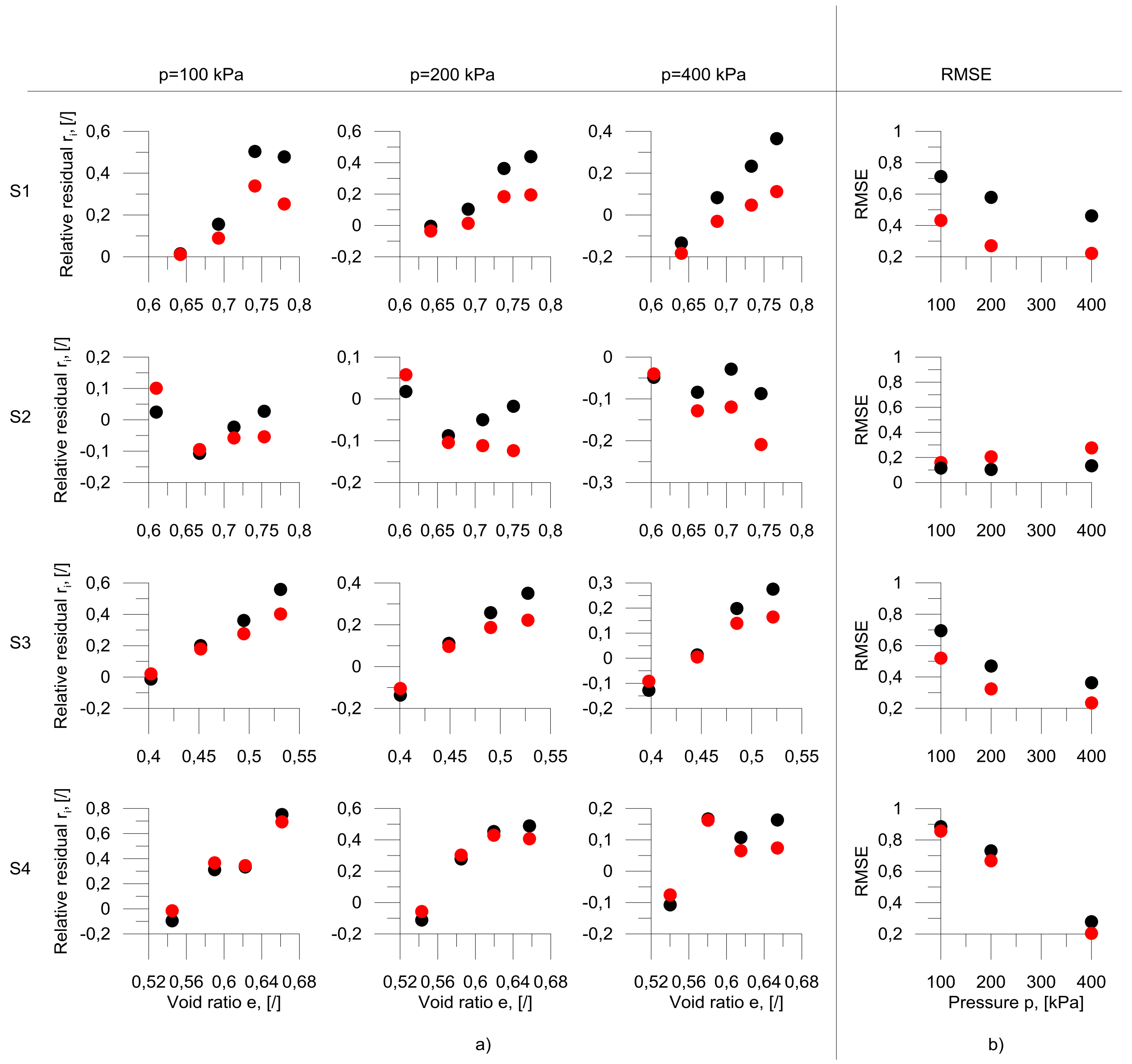

Figure 5. a) Relative errors as function of void ratio; b) RMSE as function of confining pressure. Measured data compared to predicted with Hardin constants for angular grain shape (black) and Wichtmann constants (red)

Figure 5 presents the relative residuals between the predictions of Hardin and Wichtmann and the measured data of every sand, at all three pressured and four densities S1, S3 and S4 show for both predictions an increase in relative residual with the void ratio for every stress level. In contrast S2 shows no trend in the residuals. RMSE decreases for both predictions with increasing confining pressure for samples S1, S3 and S4, while it is almost constant or even slightly increasing for S2.

For samples S1, S3 and S4 both equations overpredict measured G_{max} for specimens prepared with relative densities D_r=50-80%, while G_{max} for D_r=90-95% is predicted correctly or slightly underpredicted especially at higher confining stress. The sample S2, however, fits well with the empirical equations with slight tendency to underestimation, more so by the prediction after Wichtmann.

The residual distribution is not random; this suggests that either the equation itself may be inapplicable to describe the dependency of the G_{max} on the void ratio, or that the coefficients depend on more factors than the coefficient of uniformity.

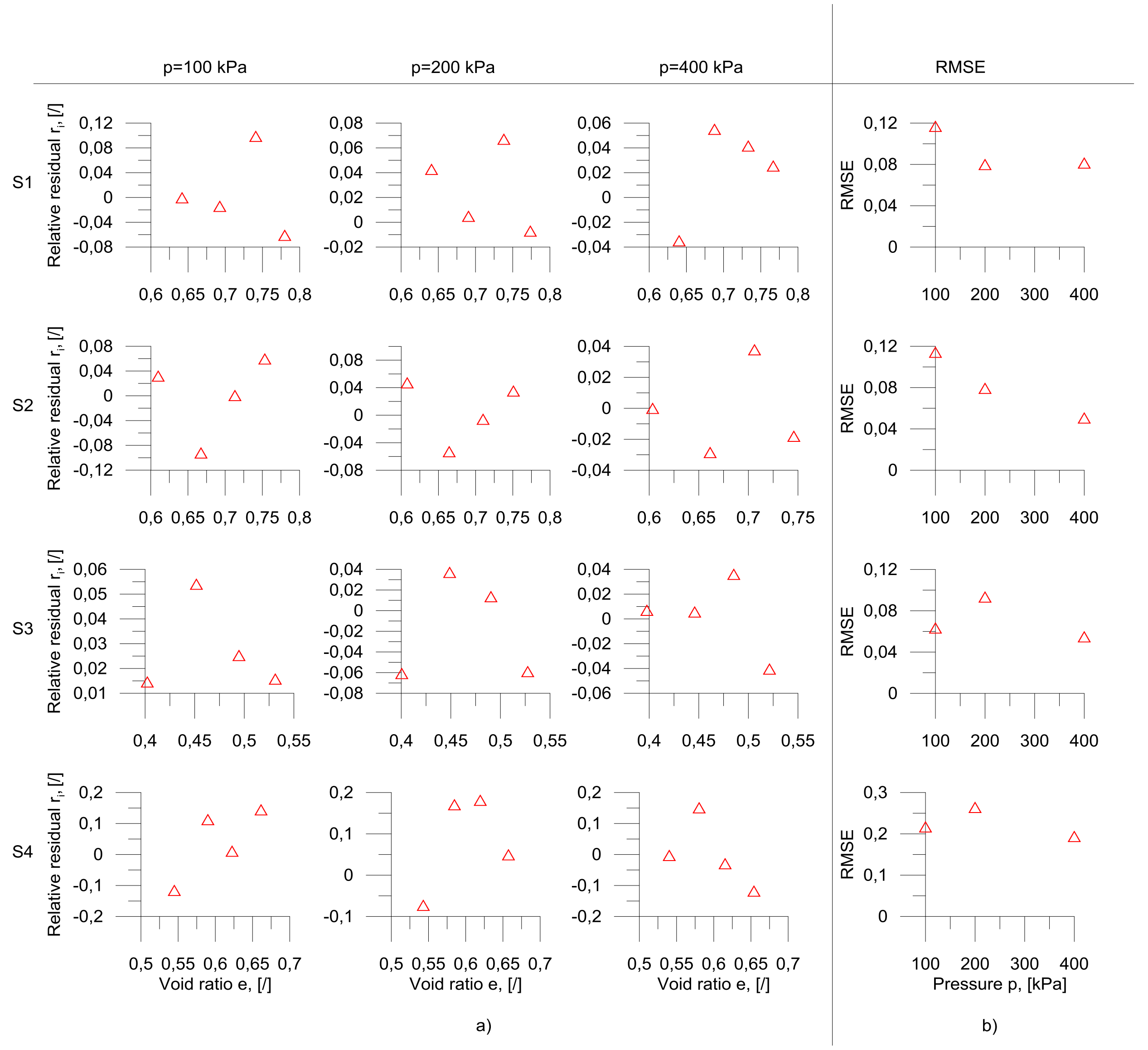

Figure 6. a) Relative errors as function of void ratio; b) RMSE as function of confining pressure. Measured data compared to predicted by Equation (1) with coefficients from Table 6

3.4 Fitting of material constants

For each sand a set of empirical coefficients was fitted to Equation (1) in order to check the general applicability of this equation type (Table 6). The 12 experimental points of four relative densities at three stress stages for each sand provide some statistical significance, the residuals and RSMEs are given in Figure 6. As expected the relative residuals are very low, however in contrast to residuals shown on Figure 5 the new residuals are distributed randomly and do not exhibit any dependency on the void ratio. Even though a slight decrease of RMSE with the stress level is still evident, the (1) type of equation seem to be applicable for the G_{max} prediction and give a good fit if the empirical coefficients $A, a,$ and n have been adjusted for each type of sand.

Table 6. Fitted empirical coefficients

Sand	A	a	n
S1	36,215	1,206	0,586
S2	1,847	3,511	0,531
S3	55,004	0,931	0,574
S4	44,778	1,070	0,586

Influence of the grain shape on the empirical coefficients is shown in Figure .

There is no apparent dependency except for a possible increase of the coefficient A with percentage of rounded particles and decrease of coefficient a again with percentage of rounded particles. In this case it could seem that the shear modulus depends on the percentage of rounded particles within the sample. Considering four distinct but known coefficients the number of their permutations is equal to 4!. Therefore the probability they will line up coincidently in ascending or descending order is equal to 1/24, while the probability of a monotone sequence in one trial is 1/12. Since 2 monotone sequences were obtained in 12 trials the probability that this dependency is coincidental amounts to 19.2%. This probability is high but not high enough to fully discard the presumption. To increase the statistical relevance of this statement, more samples have to be tested.

4 DISCUSSION

Empirical equations proposed by Hardin and Richart (1963) and Wichtmann & Triantafyllidis (2009) were compared with G_{max} measured at four sand samples from the North Sea and the plain of Northern Germany. In the case of prediction by Hardin & Richart (1963), 37.5% of predicted values differ by more than 20% from the measured data. Accounting for the coefficient of uniformity in the prediction after Wichtmann & Triantafyllidis (2009) improves the agreement, so only 25% of predicted values differ by more than 20% from the measured data. However, it does not achieve the quality reported in Wichtmann & Triantafyllidis (2014) where this prediction applied on various grain size distributions deviates more than 20% from the measured data in only 7% the cases.

Comparisons among S1 and S2 demonstrate that changes in the coefficient of uniformity alone cannot lead to the observed differences in the small strain stiffness. All samples were freshly pluviated in the laboratory and subjected to the same testing procedure, therefore the geological history, aging and cementation cannot appreciably affect the

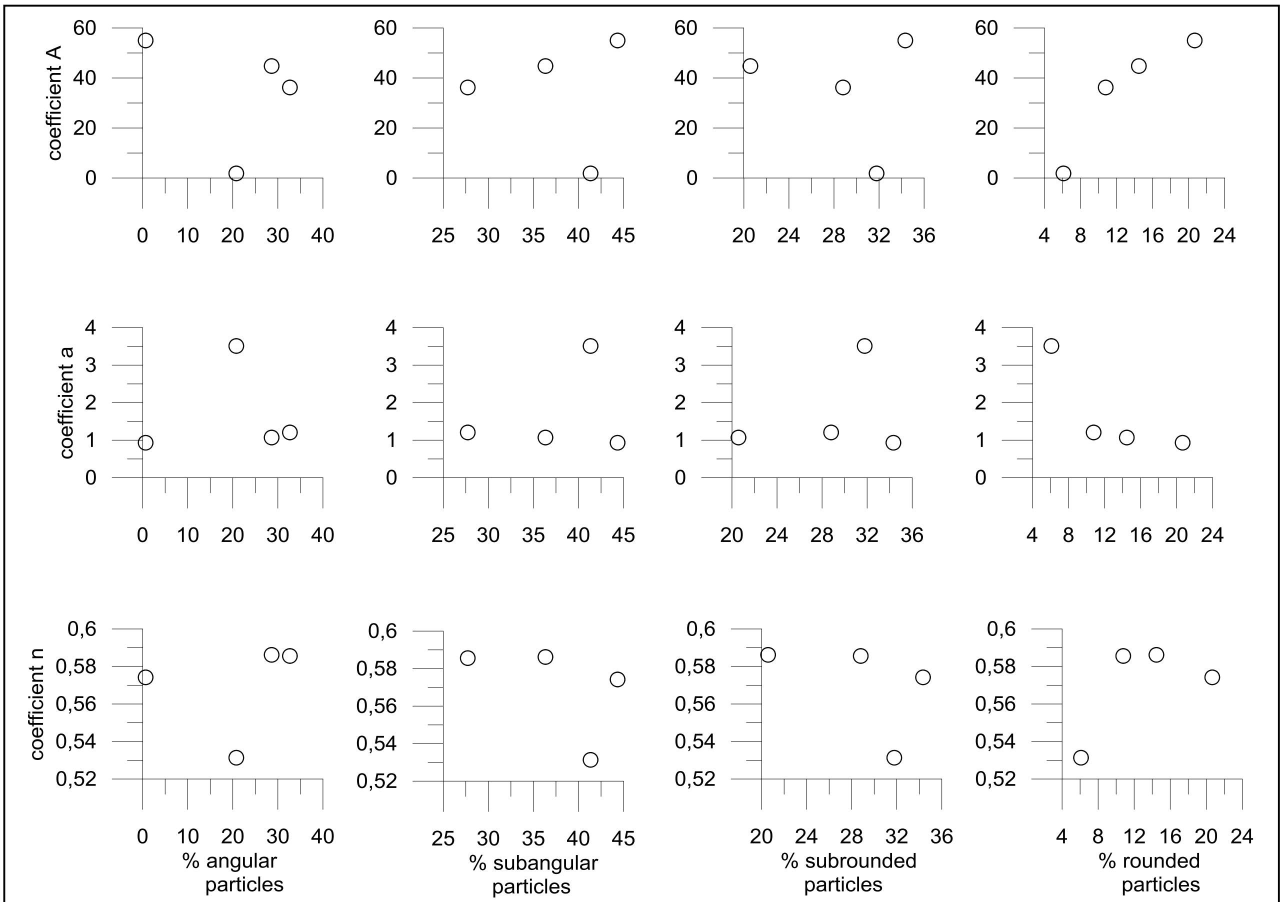

Figure 7. Influence of the grain shape on the empirical constants

results. Grain shape analysis shows that sample S2 contains the minimum percentage of rounded particles. This study seems to indicate that an increased percentage of rounded particles may lead to a decrease in the small strain stiffness for the same void ratio and isotropic confining stress.

On the one hand this observation follows the study of Shin & Santamarina (2013), who mixed rounded and angular natural and crushed sand particles at selected mass fraction and found an increase in small strain shear modulus as the mass fraction of angular particles increases. The experiments were carried out in oedometer instrumented with bender elements. On the other hand, no direct dependency on the amount of angular particles was found. Cho et al (2006), Santamarina & Cascante (1998), and Otsubo et al. (2015), however, conclude that a decrease in particle roundness or sphericity leads to a decrease in the small strain stiffness. The methods applied differs for each study: Cho et al. (2006) carried out oedometer tests on natural and crushed sands, Otsubo et al. (2015) investigated artificially roughened beads in true triaxial isotropically consolidated tests with bender elements, and Santamarina & Cascante (1998) conducted resonant column tests on rusted balls.

It was also found in this study that the deviations of the measured data from the empirical equations are more pronounced at low mean stress and low relative densities. Repeatability tests also reveal less deviation between measurements at higher stress levels. Based on Cho et al (2006) argumentation that the small strain shear stiffness deformations localize on interparticle contacts we assume that a decreasing RMSE with increasing confining stress should also be caused be the influence of the grain shape. This is in agreement with findings by Yimsiri and Soga (2000) and Otsubo et al. (2015) that the difference between rough and smooth samples gradually reduces as confining stress increases. As the confining stresses applied by Otsubo et al (2015) do not exceed 500 kPa, no or minimal grain crushing can be assumed.

Visual observation of the grain shape is somewhat subjective and operator-depended, therefore a direct comparison in terms of grain shape with other laboratories is not straightforward. Cho et al (2006) and Shin & Santamarina (2013) apply the sphericity-roundness-roughness charts for visual inspection, Otsubo et al. (2015) the optical interferometry and in this study roundness charts after Powers (1953) were applied.

The statistical relevance of the presumption is found to be of a magnitude that will neither accept nor discard it. To make a clear conclusion the number of tests, especially with soils containing higher percentage of rounded particles, has to be increased and the description of the grain shapes has to be made comparable with other studies.

As no quantity exists to describe the overall sample roughness it is possible that the combinations of subrounded and subangular particles may influence the small strain stiffness if their percentage is significant in comparison to “truly” round and angular particles. Since the empirical equations tend to overestimate the Gmax for Dr=50-80% they should be used with caution in these cases.

5 CONCLUSIONS

The empirical equations by Hardin & Richart (1963) or Wichtmann & Triantafyllidis (2009) do not always adequately describe the small strain properties of four investigated sands from the North Sea. Deviation of measured data from empirical equations was shown statistically. In case of prediction by Wichtmann & Triantafyllidis (2009), 25% of predicted values differ more than 20% from the measured data. Both Hardin& Richart and Wichtmann & Triantafyllidis empirical equations tend to overestimate the measured results up to 60% at low confining stresses and up to 40% at high confining stresses for medium and dense sands (D_r=50-80%) and underestimate for very dense sands (D_r=95%). However, the form of the void ratio function in Equation (1) has been proven to be satisfactory in describing the dependency of G_{max} on the void ratio. The coefficients of the

Equation (1) have to be adjusted to match the laboratory measurements.

It was found that the percentage of the rounded particles may influence the small strain stiffness of investigated sands, but due to the few samples tested in this study, the statistical relevance of this has not been proven. Grain shape is difficult to describe mathematically and human factors may affect the results of visual determination by charts. This must be improved in order to more closely evaluate the effect of the grain shape.

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