

Reliability-based design of a monopile foundation for offshore wind turbines based on CPT data

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ABSTRACT

A reliability-based design optimization (RBDO) of a monopile foundation for offshore wind turbines is conducted to account for the uncertainties in the design process. The RBDO aims at optimizing the cost of construction, installation and failure with respect to the ultimate limit state of a monopile foundation while accounting for the effects of uncertainties in soil parameters and lateral loads. The soil parameters are interpreted via a probabilistic link which is composed of a random field model of CPTU measurements and the transformation uncertainty relating soil parameters to CPTU measurements. The maximum likelihood method is employed to estimate the random field parameters of CPTU measurements. Probabilistic models for soil parameters and lateral loads are coupled with the nonlinear p-y finite element model to predict the response of a monopile foundation. The RBDO problem is solved by coupling the Subset Simulation reliability method with the Simulated Annealing stochastic optimization algorithm.

Keywords: *reliability, optimization, RBDO, CPT, random field*

1 INTRODUCTION

Geotechnical designs are subjected to uncertainties originating from various sources (e.g., inherent soil variability, measurement errors, transformation uncertainty, modelling assumptions). In geotechnical practice it is common to evaluate the effects of uncertainties on a design with a semi-probabilistic methodology commonly known as the partial factor of safety approach. In the partial factor of safety approach, the uncertainties in the design are quantified implicitly by partial safety factors, as defined in several design codes (e.g., Eurocode). Alternative to the factor of safety approach is the reliability-based design which explicitly accounts for the effects of uncertainties in a design. The application of the reliability-based design is considered as advantageous because it provides an insight in the likelihood of failure as a probabilistic measure (Fenton & Griffiths, 2008). In this study, the application of the reliability-based design is coupled with optimization in a procedure commonly referred to as

reliability-based design optimization (RBDO). The goal of the RBDO is to optimize performance criteria of a geotechnical design (e.g., design cost) while explicitly accounting for the effects of uncertainties.

In the context of the offshore wind industry, the application of the RBDO is beneficial due to standardized structural components (e.g., tower, monopile). However, in the majority of studies, deterministic optimization is considered. For example, Uys, Farkas, Jarmai, and Van Tonder (2007) optimized the weight and the cost of an offshore wind turbine tower. Negm and Maalawi (2000) applied the interior penalty algorithm to optimize the natural frequencies of the wind turbine. Fischer et al. (2012) examined the advantages of design optimization design by minimizing the weight of the wind turbine tower and the monopile. Sørensen and Tarp-Johansen (2005) conducted an RBDO to minimize the inspection, construction, maintenance, and failure costs of a wind turbine tower with a gravity-based foundation.

This study performs an RBDO of a monopile foundation to minimize the design costs with

respect to a reliability constraint. The design cost is expressed as a function of the monopile diameter, wall thickness, embedded length, and random parameters (i.e., lateral load and soil parameters). Special attention was given to the uncertainties associated with the interpretation of soil parameters relevant for the design of monopile foundations from CPTU measurements. The RBDO is implemented by coupling the Simulated Annealing (SA) optimization method with the Subset Simulation (SS) reliability method.

2 MONOPILE FOUNDATIONS FOR OFFSHORE WIND TURBINES

2.1 Sheringham Shoal Offshore Wind Farm

The RBDO of a monopile foundation from the Sheringham Shoal Offshore Wind Farm (SSOWF) is performed in this study. The SSOWF is an offshore wind farm located in the North Sea, 20 km north of the Norfolk coast, offshore UK. Based on the geotechnical conditions at the site and the conducted site investigations three main soil units are identified at the site, as presented in Figure 1. The Bolders Bank Formation (BDK) is a soil unit found at the seabed. The BDK formation is primarily composed of stiff clay with sand and gravel pockets. The Egmond Ground Formation (EG) is a soil unit found below the BDK formation. The EG formation is a mixture of dense to very dense sands. The Swarte Bank formation (SBK) is a soil unit found below the EG formation. The SBK formation is a mixture of hard silty clay with occasional marine interglacial sediments (Saue & Meyer, 2009).

2.2 Numerical pile-soil model

Given that the monopile foundations for offshore wind turbines are dominantly loaded laterally, the response of a monopile foundation is commonly simulated by the p-y method (e.g., DNV, 2010). The p-y method models the response of the soil domain to the lateral loading of the pile by a series of nonlinear soil springs. The material behavior of the soil springs is defined by the p-y curves which are developed for different soil

types (e.g., clay, sand) and loading conditions (i.e., static, cyclic). In this study, the static p-y curves for stiff clay are implemented to simulate the soil response of the BDK and the SBK soil units. On the other hand, the static p-y curves for sand are implemented to evaluate the response of the EG soil unit.

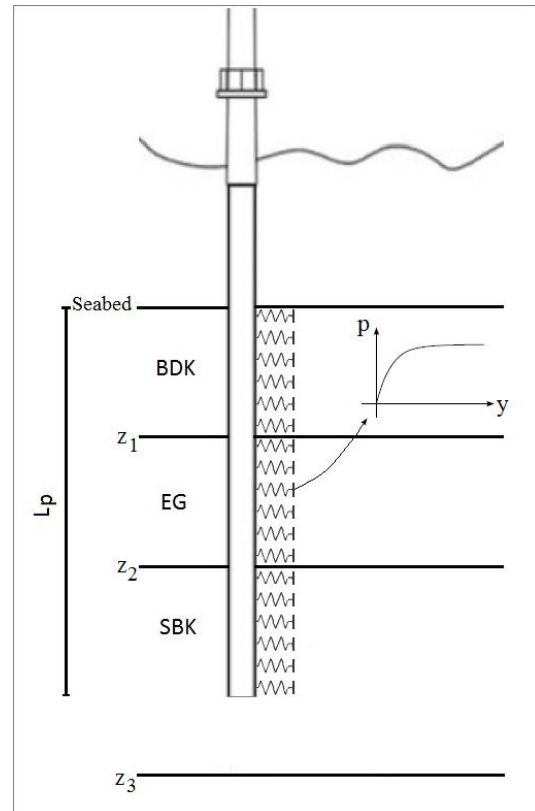


Figure 1: Three main soil units at the SSOWF site.

In addition to several empirical parameters and the soil unit weight, the p-y curves for stiff clay are a function of undrained shear strength, s_u , while the p-y curves for sand are a function of the friction angle ϕ . Given that soils exhibit spatial variability due to the randomness of geological processes involved in the creation of the soil formations at the SSOWF, the spatial variability of s_u and ϕ and their effect on the monopile response will be examined. Other parameters of the p-y curves are considered to be deterministic because of a relatively low variability (e.g., unit weight) or due to their empirical and model dependent origins.

The monopile material is steel with density of $\rho_s=7800 \text{ kN/m}^3$, and elastic behavior defined by Young's modulus of $E_p=2.1 \times 10^5 \text{ MPa}$ and Poisson's ratio $\nu_s=0.3$.

3 PROBABILISTIC SOIL PARAMETER INTERPRETATION FROM CPT DATA

The derivation of soil parameters for the design of monopile foundations relies primarily on the data obtained from the CPTU profiles at the locations of the planned wind turbines and several boreholes at the site. Soil samples extracted from the boreholes are commonly used to calibrate the relations between the CPTU measurements and soil parameters such that the soil parameters can be estimate at the planned locations of the wind turbines. A probabilistic interpretation of s_u and ϕ based on CPTU data is examined in this study. The probabilistic interpretation includes the inherent variability of the CPTU measurements and the transformation uncertainty associated with the relations between and s_u and ϕ and the CPTU measurements.

3.1 s_u interpretation

The relation between the CPTU measurements and s_u can be established as follows (e.g., Lunne, Robertson, & Powell, 1997):

$$s_u = \frac{q_t - \sigma_{v0}}{N_k} \quad (1)$$

where q_t (MPa) is the corrected cone tip resistance, σ_{v0} (kPa) is the in-situ overburden pressure, while N_k is the empirical cone factor. The parameters of the right side of Eq. 1 are associated with uncertainties. The uncertainties in q_t are a result of both the inherent soil variability and the measurement error. Although σ_{v0} can be influenced by various sources of uncertainty, a deterministic stress state is assumed, dependent on the soil unit weight, soil depth and water level. Given its empirical nature N_k is associated with uncertainties. A study on the values of N_k in Kulhawy, Birgisson, and Grigoriu (1992) reveals that the uncertainty of N_k depends on the soil test used to estimate s_u . Table 1 presents the mean and CoV values

of N_k for different soil tests (Kulhawy et al., 1992):

Table 1: Mean and CoV values of N_k for different s_u tests from Kulhawy et al. (1992).

s_u test	Mean	CoV
CIUC	12.674	35
UU	19.531	29
VST	11.038	40

CIUC, consolidated isotropic undrained triaxial compression test, UU unconsolidated undrained triaxial compression test, VST vane shear test.

The derivation of s_u values from q_t measurements is based on the assumption that q_t measurements are an outcome of a lognormal random field. The corresponding lognormal random field of q_t is defined by a deterministic trend, μ_{q_t} , standard deviation, σ_{q_t} , and the correlation length, θ_{q_t} .

The parameters of the q_t random field are estimated with the maximum likelihood method from the values of the corresponding normal random field $\ln q_t$ by assuming the exponential correlation model (e.g., Fenton & Griffiths, 2008). Given the maximum likelihood estimates of the mean, $\mu_{\ln q_t}$, standard deviation, $\sigma_{\ln q_t}$, and the correlation length, $\theta_{\ln q_t}$, the parameters of the lognormal random field can be derived as follows.

$$\mu_{q_t} = \exp\left(\mu_{\ln q_t} + \frac{\sigma_{\ln q_t}^2}{2}\right) \quad (2a)$$

$$\sigma_{q_t} = \mu_{q_t} \sqrt{\exp(\sigma_{\ln q_t}^2) - 1} \quad (2b)$$

N_k is assumed to be lognormally distributed with a site-specific mean, μ_{N_k} , with the CoV_{N_k} as reported in Table 1. Given that σ_{v0} is deterministic, and both q_t and N_k are lognormally distributed the natural logarithm transformation of Eq. 1 can be utilized to derive the parameters of s_u :

$$\ln s_u = \ln(q_t - \sigma_{v0}) - \ln N_k \quad (3)$$

Since q_t is lognormally distributed and σ_{v0} is a deterministic value, $(q_t - \sigma_{v0})$ is lognormally distributed with the following parameters:

$$\mu_{(q_t - \sigma_{v0})} = \mu_{q_t} - \sigma_{v0}; \quad \sigma_{(q_t - \sigma_{v0})} = \sigma_{q_t} \quad (4)$$

Since both terms in the right side of Eq. 3 are normally distributed, $\ln s_u$ is normally distributed with the following parameters:

$$\mu_{\ln s_u} = \mu_{\ln(q_t - \sigma_{v0})} - \mu_{\ln N_k} \quad (5a)$$

$$\sigma_{\ln s_u}^2 = \sigma_{\ln(q_t - \sigma_{v0})}^2 + \sigma_{\ln N_k}^2 \quad (5b)$$

where $\mu_{\ln(q_t - \sigma_{v0})}$ and $\sigma_{\ln(q_t - \sigma_{v0})}$ are the parameters of $\ln(q_t - \sigma_{v0})$, while $\mu_{\ln N_k}$ and $\sigma_{\ln N_k}$ are the parameters of $\ln N_k$. The parameters of $\ln(q_t - \sigma_{v0})$ are calculated as follows:

$$\sigma_{\ln(q_t - \sigma_{v0})} = \sqrt{\ln \left(1 + \frac{\sigma_{(q_t - \sigma_{v0})}^2}{\mu_{(q_t - \sigma_{v0})}^2} \right)} \quad (6a)$$

$$\mu_{\ln(q_t - \sigma_{v0})} = \ln \mu_{(q_t - \sigma_{v0})} - \frac{1}{2} \sigma_{\ln(q_t - \sigma_{v0})}^2 \quad (6b)$$

The parameters of $\ln N_k$ are calculated as follows:

$$\sigma_{\ln N_k} = \sqrt{\ln \left(1 + \frac{\sigma_{N_k}^2}{\mu_{N_k}^2} \right)} \quad (7a)$$

$$\mu_{\ln N_k} = \ln \mu_{N_k} - \frac{1}{2} \sigma_{\ln N_k}^2 \quad (7b)$$

It is assumed that the transformation does not affect the autocorrelation properties, $\theta_{s_u} = \theta_{q_t}$. Figure 2 present a realization of an s_u random field based on the following maximum likelihood estimates of a lognormal random field fitted to a CPTU at the SSOWF site; $\mu_{q_t} = 1.9$ MPa, $\sigma_{q_t} = 0.6$ MPa, and $\theta_{\ln q_t} = 1$. For the BDK and SBK soil units the values of $\mu_{N_k} = 15$ and $CoV_{N_k} = 0.35$ are selected based on the available laboratory test data from the SSOWF site (Saue & Meyer, 2009).

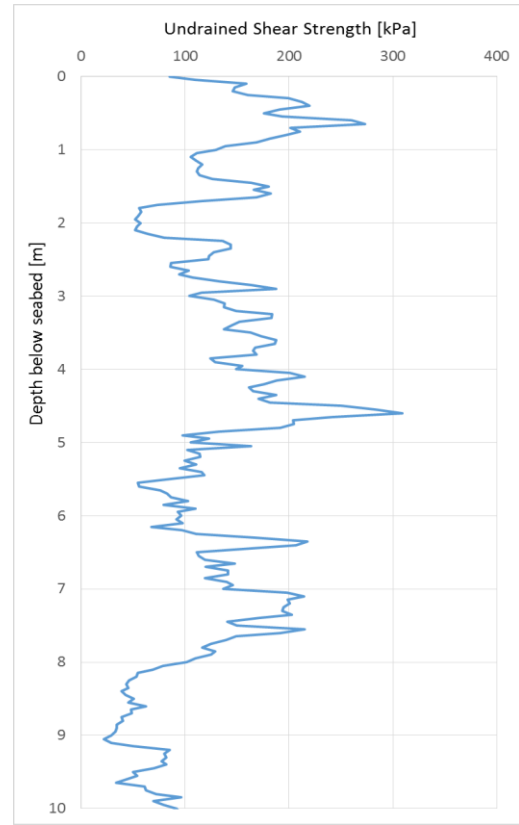


Figure 2: Example s_u profile.

3.2 φ interpretation

The value of φ is interpreted from the CPTU measurements based on the relations proposed by Robertson and Campanella (1983). The value of φ is shown to be dependent on the value of q_t scaled by the effective in-situ stresses, σ'_{v0} .

After examining the established relation between φ and q_t , as presented in Figure 3, it is observed that a relatively simple regression model can be employed to approximate the relation. The regression model has the following formulation:

$$\varphi = 5.03 \ln \left(\frac{q_t}{\sigma'_{v0}} \right) + 52.04 + \varepsilon \quad (8)$$

where ε is the transformation uncertainty which accounts for the model error and the regression model error. ε is modeled as a normal random variable with zero-mean and standard deviation $\sigma_\varepsilon = 2.8^\circ$, as reported in Kulhawy and Mayne (1990). It is important to note that the regression model is valid only for the range of φ presented in Figure 3, $30^\circ \leq \varphi \leq 50^\circ$.

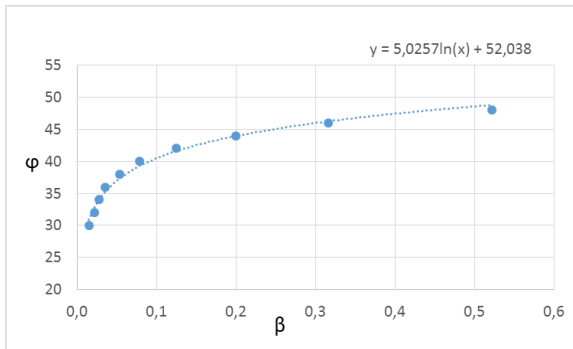


Figure 3: Relation between $\beta=q_t/\sigma'_{v0}$ and φ .

The statistical parameters of φ are derived by assuming a deterministic stress profile σ'_{v0} and that q_t measurements are an outcome of a lognormal random field. As discussed in Section 3.1, the parameters of the lognormal random field can be estimated with the maximum likelihood method.

Given that q_t is lognormally distributed while ε is normally distributed it follows from Eq. 8 that φ is normally distributed with the following parameters:

$$\mu_\varphi = 5.03 \left(\mu_{\ln q_t} - \ln \sigma'_{v0} \right) + 52.04 \quad (9a)$$

$$\sigma_\varphi^2 = 5.03^2 \sigma_{\ln q_t}^2 + \sigma_\varepsilon^2 \quad (9b)$$

where $\mu_{\ln q_t}$ and $\sigma_{\ln q_t}$ are the mean and standard deviation of $\ln q_t$ which are estimated with the maximum likelihood method. It is assumed that the transformation does not affect the autocorrelation properties, $\theta_\varphi = \theta_{q_t}$.

An example random field realization of φ is present in Figure 4.

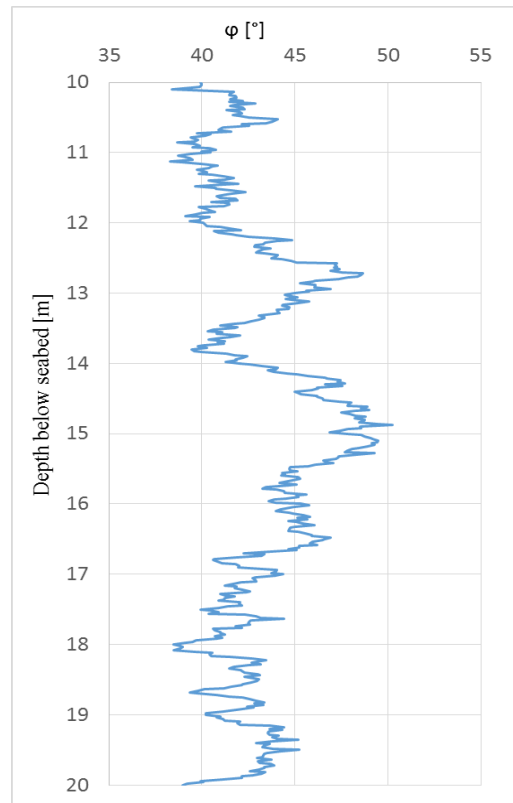


Figure 4: Example φ profile.

4 RANDOM LOAD

The monopile in this study is loaded laterally with a load composed of a horizontal force H and a bending moment $M=H \cdot 30$ m. H is assumed to be random and distributed according to the Gumbel distribution with the mean $\mu_H=2500$ kN and $\text{CoV}_H=0.2$.

5 RBDO OF A MONOPILE FOUNDATION

The RBDO of a monopile foundation is conducted to minimize the total cost with respect to a set of monopile design parameters $\mathbf{t}=[D, w, L_P]$, where D (m) is the monopile diameter, w (m) is the monopile wall thickness, and L_P (m) is the monopile length. The optimization is conducted in the discretized space of random parameters such that $D \in [4, 4.1, \dots, 7]$, $w \in [0.03, 0.04, \dots, 0.1]$, and $L_P \in [25, 26, \dots, 40]$. The design cost of a monopile foundation is approximated by a cost of $C_i=2\text{€}/\text{kg}$ of the monopile weight, while the expected failure cost is assumed to be $C_F=10^7\text{€}$.

The RBDO problem is defined as follows:

minimize: $C(\mathbf{x}, \mathbf{t}) =$

$$C_i L_p \rho_s \pi \left[\left(\frac{D}{2} \right)^2 - \left(\frac{D}{2} - w \right)^2 \right] \quad (10a)$$

$$+ C_F P_F(\mathbf{x}, \mathbf{t})$$

subject to:

$$[4, 0.03, 25] \leq \mathbf{t} \leq [7, 0.1, 40] \quad (10b)$$

$$P_F(\mathbf{x}, \mathbf{t}) \leq 10^{-4} \quad (10c)$$

where $\mathbf{x} = [H, s_u, \varphi]^T$ is a vector of random parameters composed of H, s_u random fields for the BDK and SBK soil units, and φ random field for the EG soil unit, $P_F(\mathbf{x}, \mathbf{t})$ is the probability of exceeding the ultimate limit state for a given combination of design parameters.

In this study, the ultimate limit state of a monopile foundation is defined by the yield strength of the monopile steel, $\sigma_{lim} = 235$ MPa. The ultimate limit state can be defined by the corresponding performance function:

$$g(\mathbf{x}', \mathbf{t}') = \sigma_{lim} - \sigma(\mathbf{x}', \mathbf{t}') \quad (11)$$

where $\sigma(\mathbf{x}', \mathbf{t}')$ is the maximum stress in the monopile for a given combination of \mathbf{x}' and \mathbf{t}' . The probability of exceeding the ultimate limit state, for a given combination of design parameters \mathbf{t}' , $P_F = P(g(\mathbf{x}, \mathbf{t}') \leq 0)$ is calculated with the SS method (Au & Beck, 2001). The SS is an efficient and robust reliability method which expresses the reliability problem by a series of intermediate conditional reliability problems. The conditional reliability problems correspond to, prior to the analysis unknown, series of decreasing intermediate failure limits. The SS method provides efficient performance by specifying the probabilities of the conditional reliability problems sufficiently large (e.g., $P=0.1$) so that they can be evaluated with a relatively low number of samples of random parameters.

The reliability of a monopile foundation is evaluated with the SS method by defining the probabilities of the intermediate conditional

reliability problems to be $P=0.1$. The conditional probabilities are evaluated with 200 samples of random parameters.

The RBDO of a monopile foundation is conducted by coupling the SA method and the SS method to solve the optimization and reliability problems, respectively. The SA is a stochastic optimization method applied for discrete and continuous optimization problems. The SA method is selected due to its robust algorithm which is capable of avoiding local minima in search of the global minimum (e.g., Spall, 2005). In order to integrate the reliability constraint in Eq. 10c into the optimization process, the algorithm of the SA method is adapted such that $C(\mathbf{x}, \mathbf{t}) = \infty$ in case of reliability constraint violation, $P_F(\mathbf{x}, \mathbf{t}) > 10^{-4}$.

6 RESULTS

The SA optimization is initiated with following values of the design parameters: $\mathbf{t}' = [5.5, 0.05, 30]$, with $P_F(\mathbf{x}, \mathbf{t}') < 10^{-4}$ and $C(\mathbf{x}, \mathbf{t}') = 4.03 \times 10^5 \text{€}$.

The SA algorithm was employed with 1000 iterations to locate the minimum design costs. For each iteration of the algorithm, the probability of exceeding the ultimate limit state of the monopile is evaluated with the SS method. The SS method is implemented with 200 simulations of random parameters per conditional reliability problem. Since the SA method does not provide convergence criteria for the estimate of the minimal design cost, several independent optimizations are conducted to achieve a robust estimate. In total seven optimizations with the SA algorithm are conducted, as presented in Table 2.

Table 2: RBDO results

C [10^5€]	w [m]	L_p [m]	D [m]	P_F [10^{-6}]
2.55	0.04	26	5.0	8.5
2.60	0.04	26	5.1	4.6
2.64	0.04	27	5.0	8.5
2.90	0.04	27	5.7	3.3
2.70	0.04	27	5.1	8.5
2.65	0.04	26	5.2	79
2.75	0.04	26	5.4	39

Figure 5 illustrates the results from three optimizations with the SA method. It is important to note that the estimate of the optimal design cost is the minimal design cost encountered during the optimization process.

As observed from Table 2 the estimate of the minimal design cost is found between 2.55 and 2.90×10^5 €. The optimal design costs correspond to the monopile designs with w of 0.04 m, L_p 26 or 27 m and D between 5 and 5.7 m.

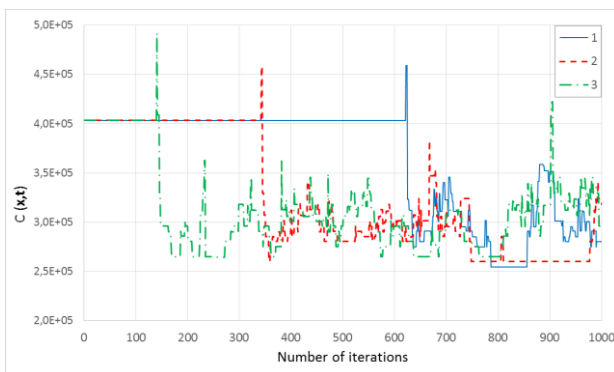


Figure 5: Design cost with the number of iterations of the SA optimization.

The results in Table 2 and Figure 6 demonstrate that the reliability constraint is implemented successfully in the SA algorithm. The reliability constraint is satisfied throughout the optimization process since $P_F \leq 10^{-4}$. The variation in the estimates of P_F in Table 2 can be attributed to the stochastic nature of the SS method.

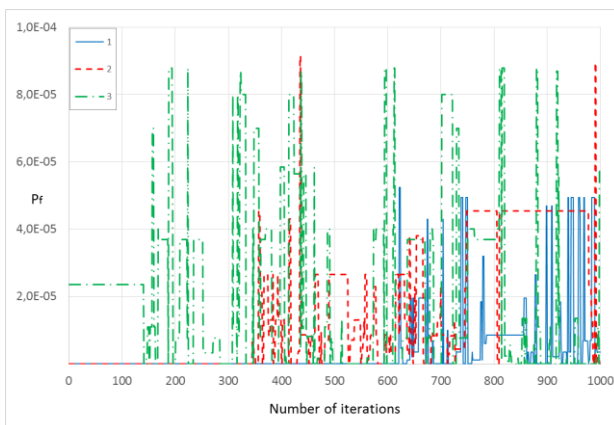


Figure 6: P_F with the number of iterations of the SA optimization.

7 CONCLUSION

This study presented an RBDO of a monopile foundation for offshore wind turbines. The goal was to minimize the design cost which includes the cost of production, installation and failure. The RBDO is conducted with the Simulated Annealing optimization method and the Subset Simulation reliability method. The Subset Simulation method is implemented to evaluate the probability of exceeding the ultimate limit state of a monopile foundation with respect to the uncertainties in the soil parameters and lateral loads.

Special attention was given to the interpretation of soil parameters relevant for the design of monopile foundations from CPTU measurements. A probabilistic interpretation of undrained shear strength and friction angle based on CPTU measurements is implemented to integrate the inherent soil variability of the cone tip resistance with the transformation uncertainty associated with the corresponding soil parameter.

This study demonstrated that the reliability-based design optimization provides a consistent framework for dealing with uncertainties in the design of monopile foundations.

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