# Numerical models on anisotropy of rocks

Henok M. Kassa
*Statoil ASA, Norway, hmka@statoil.com*

Steinar Nordal
*Norwegian University of Science and Technology, Norway, steinar.nordal@ntnu.no*

ABSTRACT
*Most rocks found in nature are inherently anisotropic, which exhibit a variation in mechanical properties in different directions. But due to formation processes rock that we encounter are Transversely Isotropic, which has the same property in one plane and different properties in directions normal to this plane.*

*To understand and capture the behavior of such rocks, scholars have proposed models (criteria), which are either strength or stiffness dependent. But in this paper among the many criteria proposed so far, only some representatives have been discussed. These criteria are classified in to three groups; mathematically continuous criteria, empirical continuous criteria and weakness plane based criteria. Experimental data have been extracted from literatures; Such as, F. A, Donath 1964, a triaxial data on Martinsburg slate. Strength Anisotropy is used as a main parameter to evaluate the anisotropy of rocks. Then a comparison is performed between the experimental data and the selected failure criteria.*

*In addition, numerical simulation for layered rock system, which can represent transversal isotropic behavior, has been conducted using a commercially available finite element code PLAXIS. And the result was compared with experimental data for artificially prepared layered rocks, Yi-Shao Lai, 1999.*

**Keywords: Anisotropy, Bedding plane, Failure criteria, Plaxis.**

## 1 INTRODUCTION

Geo-materials, including rock and soil, are often highly anisotropic, i.e., their properties vary with direction. A material is considered anisotropic if its strength properties are dependent on the direction of the applied stresses. For instance, the elastic stiffness in one direction may be more than double that in another direction. In many engineering application anisotropy is neglected as it is difficult to determine the anisotropy parameters. The structure of a rock is characterized by a number of factors, which may have a particular orientation: bedding, stratification, schistosity planes, foliation, cracking, joints. Anisotropy, which is a characteristic of metamorphic rocks such as schist, slate or gneiss, is due to the existence of mineral foliation. Sedimentary rocks such as sandstone, shale and limestone may be anisotropic, as a result of stratification. On a larger scale, any rock mass may be crosscut by one or more families of discontinuities. In this paper, we focus on a type of rocks with a particular anisotropy which is interesting in that it combines mineral orientation and cracking.

In general the objective of this paper is to evaluate the effect if anisotropy on the strength and deformation characteristic of transversely isotropic rocks, where linear type of anisotropy is dealt in detail. Furthermore, different proposed anisotropic failure criteria have been discussed and used to estimate the failure strength of transversely isotropic rocks and compare is with experimental results and simulation results from a 2D commercial FEM program, PLAXIS.

### *1.1 Anisotropy in Rocks*

Rock anisotropy is a well-known behavior and is of considerable interest in the field of rock mechanics and engineering. This behavior related to both transport and mechanical properties is highly dependent on the sampling orientation with respect to loading directions.

Natural soils or rocks exhibit two common types of anisotropy in stiffness: *inherent* and *stress induced*

Several types of rocks, such as metamorphic and sedimentary rocks, have inherent or structural anisotropy. Among sedimentary rocks, the most widespread are shale, siltstone and clay stone. These rocks, which are formed by deposits of clay and silt sediment, exhibit strong inherent anisotropy, manifesting itself in a directional dependence of deformation characteristics. The anisotropy is strongly related to the microstructure, in particular the existence of bedding planes which mark the limits of strata and can be easily identified by a visual examination. The study of the mechanical behavior of sedimentary rocks, especially shale and mudstone, is of particular interest to the oil exploration industry as well as to civil and mining engineering.

An induced anisotropy is common in granular soil mass when the materials re-orientation and re-arrangement occurs under stress orientation. Induced anisotropy is directly related to strain-induced particle re-orientation associated with changes in stress (Cassagrande and Carrillo, 1944).

### *1.2 Representation of Rock Anisotropy*

Among different testing techniques and testing methods for determining the anisotropic parameters of rocks, the most classical experiment is the conventional triaxial compression test, with various loading orientations and confining pressures. Results of the investigation are expressed in terms of strength anisotropy which can be represented using plots of stress-strain and compression strength vs. orientation angle of the bedding plane (anisotropy curve). Here the Author prefers to deal with the later type of plots as it is straight forward and can easily explain anisotropy.

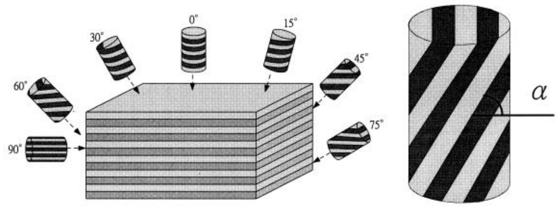



*Figure 1 Samples taken at different orientation of bedding or schistocity planes*

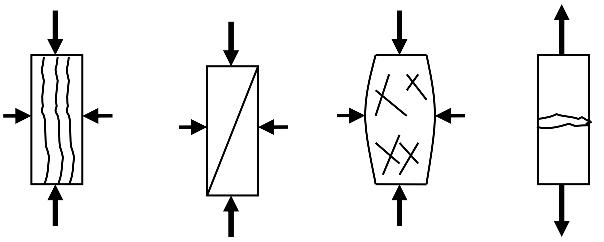
Most anisotropic geo-materials are either orthotropic or transversely isotropic materials. For a general anisotropic material, each stress component is linearly related to every strain component by independent coefficients.

$$\sigma_{ij} = \sum_{k,l} C_{ijkl} \varepsilon_{kl} \tag{1}$$

Failure of transversely isotropic rocks under uniaxial tests have been catagorized in to two types, i.e., failure of the intact rock as a whole (matrix or intact failure) and failure along the weak schistose plane (sliding type failure). Several scholars have developed failure criteria for transversely isotropic and orthotropic rocks that account for strength anisotropy with respect to orientation of bedding angle and confining pressures. Here the author has tried to discuss some of it.

## 2 FAILURE CRITERIA (MODELS) FOR ANISOTROPIC ROCKS

Analysis of a wide range of problems related to geo-materials requires knowledge of failure processes in the materials. Rock fails when the surrounding stress exceeds the tensile, compressive or the shear strength of the rock formation. There are several types of rock failure depending on rock lithology, rock microstructures and applied confining stresses.



(a) (b) (c) (d)

*Figure 2 Rock failure types (a) Splitting, (b) Shear failure, (c) Multiple shear fracture, (d) Tensile Failure*

Over the years, a number of failure criteria, which can reasonably estimate the stresses and strength of anisotropic geo-materials have been proposed. The basic frame work of these criteria is derived from isotropic homogeneous bodies.

Failure criteria can be classified as: stress-based and non-stress based types. Stress based failure criteria are mostly for brittle or ductile materials. It is completely dependent on the stresses acting on the material. In order to define such kinds of criteria we need to perform a hand full of tests (such as: uniaxial tension/ compression), whereas, in a Non-stress based criteria, whether a material succeeds or fails may depend on other factors such as stiffness, fatigue resistance, creep resistance, etc.

The various failure criteria for anisotropic materials, based on assumptions and techniques used can also be classified in to three main groups (G. Duveau etal, 1998):

I) Mathematical continuous approach: These criteria consider continuous body with a continuous variation of strength. Mathematical techniques and material symmetries are used to describe anisotropy in strength. W.G. Pariseau, 1972, proposed a model that can be categorized in this approach. The model is basically a modification of Hill Criterion, except that it accounts the strength difference in tensile and compressive loading and the dependency of strength on the mean stress.

II) Empirical continuous criteria: It involves determination of variation laws as a function of the loading orientation for some materials parameters used in an isotropic criterion. The variation laws are fully empirical in nature and are calibrated from experimental investigation. As an example we can have a look at one of the model proposed by J.C. Jaeger, 1971.

III) Discontinuous weakness planes based theories: Criteria (models) under this category include the physical mechanisms in the failure process. Besides, it's assumed that anisotropic bodies fail either due to the fracture of the bedding planes or the fracture of the rock matrix. For instance, E. Hoek, 1983, describes distinctively on the two modes of failure (along bedding plane and rock matrix failure).

### *2.1 Single plane of weakness theory by Jaeger*

It is a discontinuous weakness plane based criteria. In this theory, the anisotropic material is seen as an isotropic body containing one set of weakness planes. The failure in the rock matrix and along weakness planes is together described by the Mohr-Coulomb type criterion. However, the values of cohesion and friction are different for rock matrix and weakness planes. Thus, the failure criterion is expressed by the following equations:

*For rock matrix failure*

$$\tau = c + \sigma \tan \phi \qquad (2)$$

*For bedding plane (weakness plane) failure*

$$\tau_\theta = c' + \sigma_\theta \tan \phi' \qquad (3)$$

Where: $\phi$ and $c$ are friction angle and cohesion of the intact rock (rock matrix) with $\tau$ and $\sigma$ are the shear and normal stress in the Mohr diagram. But in equation (3), $c'$ and $\phi'$ are the cohesion and friction angle on the weak (bedding plane) oriented at $\theta$ degree from the horizontal plane. In addition, $\tau_\theta$ and $\sigma_\theta$ are the shear and normal stresses on the weak plane.

This criterion needs four material parameters to define it that are easy to determine from a conventional triaxial test. $\phi$ and $c$ can be determined from tests conducted on a specimen with bedding plane orientation of θ=0 and θ=90, see *Figure 1*. However, many experimental results show that the strength at θ=0 is different from that of at θ=90. Therefore, it is important to determine cohesion and friction angles for the two cases. Meanwhile, the other two parameters of the model, for failure along the weak plane can be determined from triaxial tests for bedding orientations of $30^\circ \le \theta \le 45^\circ$. This is because failure along weakness plane is usually expected at these values of orientation angles.

*Table 1 Martinsburg slate tested by F.A., Donath*

| σ3(Mpa) | σ1(Mpa) | | | | | | |
|---|---|---|---|---|---|---|---|
| | θ=0 | θ=15 | θ=30 | θ=45 | θ=60 | θ=75 | θ=90 |
| 3.5 | 128 | 51 | 22 | 43 | 75 | 128 | 194 |
| 10 | 162 | 82 | 44 | 63 | 102 | 160 | 241 |
| 35 | 272 | 134 | 87 | 107 | 150 | 216 | 335 |
| 50 | 355 | 172 | 129 | 150 | 194 | 284 | 410 |
| 100 | 530 | 286 | 230 | 260 | 315 | 456 | 600 |

This criterion provides a fairly accurate simulation of experimental data for transversely isotropic materials but it requires a wide range of tests and considerable amount of curve fittings. In this paper, for a purpose of comparison, the data from Martinsburg slate tested by Donath F.A., 1964, *Table 1*, has been used.

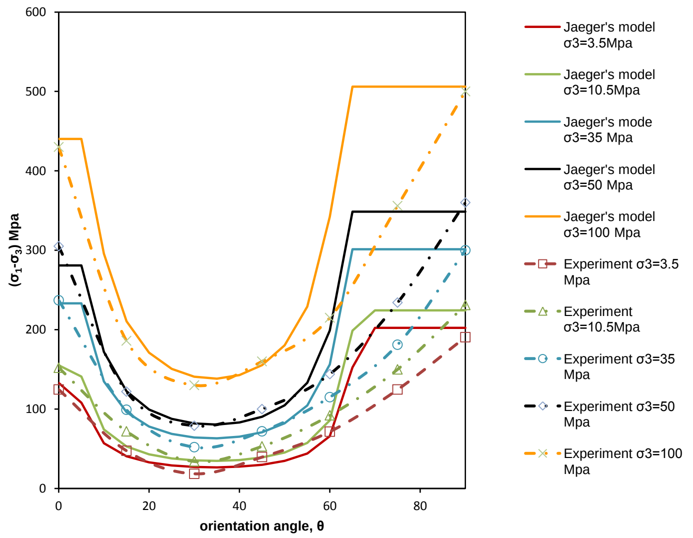



*Figure 3 Comparison of Jaeger's single plane of weakness criteria and experimental data*

Even if the distinction between the two failure mechanisms may appear too simplistic for such type of discontinuous based failure criterion, we can see good agreement between the numerical and experimental values when the bedding orientation θ<60. But it appears to be a disadvantage of this simple criteria to overestimate the strength when the bedding orientation is 60<θ<90.

## *2.2 Hoek-Brown Failure Criterion*

Based on previous studies by Jaeger, J. C. 1971, Hoek & Brown, 1980 have developed an empirical mathematical model which adequately predicts fracture propagation in rocks. In developing this empirical relationship between the principal stresses Hoek and Brown have attempted to satisfy the following conditions:

- The failure criterion should give good agreement with experimentally determined rock strength values.
- The failure criterion should be expressed by mathematically simple equations based, to the maximum extent possible, upon dimensionless parameters.
- The failure criterion should offer the possibility of extension to deal with anisotropic failure and the failure of jointed rock masses.

The process used by Hoek & Brown, 1980 in deriving their empirical failure criterion was one of pure trial and error from many experimental data. The proposed criterion is based on major and minor principal stresses.

$$\sigma_1 = \sigma_3 + (m\sigma_c\sigma_3 + s\sigma_c^2)^{0.5} \quad (4)$$

Where, $\sigma_1$ and $\sigma_3$ are major and minor principal stresses; $\sigma_c$ is the uniaxial compressive strength of the intact rock matrix. $m$ and $s$ are empirical parameters of this criterion. The constant $m$ has a positive value ranging from 0.001 for highly disturbed rock to 25 for hard intact rocks, while the value of the constant $s$ varies from 0 for jointed rocks to 1 for intact rock mass

The original criteria proposed by Hoek and Brown, 1980 equation (4) was based on isotropic rock mass. But rocks like slates and shales are schistose or layered inherently, which shows different strength on the schistocity plane to that of planes perpendicular to it. In order to determine the strength variations of such rocks in relation to the orientation of the schistocity plane , some modifications on the original theory have been added. Hoek in 1983, proposed a different approach for schistose rocks by making use of the strength variation of a rock mass containing schistosity as described by Jaeger and Cook 1969.

$$\sigma_1 = \sigma_3 + \frac{2(c_i + \sigma_3 tan\phi_i)}{(1 - tan\phi_i tan\theta)sin2\theta} \quad (5)$$

Where $c_i$ and $\phi_i$ are the instantaneous cohesion and friction on the weakness plane. $\theta$ is the orientation of bedding plane from the direction of major principal stress. Equation (5) as $\theta \rightarrow 0$ and $\theta \rightarrow 90$ gives us unrealistic values of strength, which means that the failure criteria in this region is defined by the original criteria as given in equation (4). Hoek and Brown suggested the use of equation (5) when weak plane's orientation is in the range of $\theta = \phi \pm 25$.

Equation (5) is invariably dependent on pre-determined values of the instantaneous cohesion and friction values of the weak plane. In order to determine strength parameters of a rock along its weak plane, various empirical formulations have been suggested. However, in this paper, owing to the fact that several experimental results were found to fit the curves, the relationships detailed in equation (6) – (8) have been selected.

$$\phi_i = A\text{rctan}\left[h\cos^2(30 + \frac{1}{3}\arcsin h^{-\frac{3}{2}}) - 1\right]^{-\frac{1}{2}} \quad (6)$$

Where :

$$h = 1 + \frac{16(m\sigma_n + s\sigma_c)}{3m^2\sigma_c} \quad (7a)$$

$$\sigma = \frac{\sigma_1 + \sigma_3}{2} - \frac{\sigma_1 - \sigma_3}{2} cos2\theta \quad (7b)$$

and the instantaneous cohesion can be calculated from:

$$c_i = \tau - \sigma_n \tan\phi_i \quad (8)$$

Data from the Martinsburg slate, *Table 1*, has once more been used to assess the quality of the Hoek-Brown criterion. And as can be seen from the plots, H-B model vs.

experimental data, prediction of strength variation with schistosity angle is fairly accurate where weakness plane orientation lies within 20<θ<60.

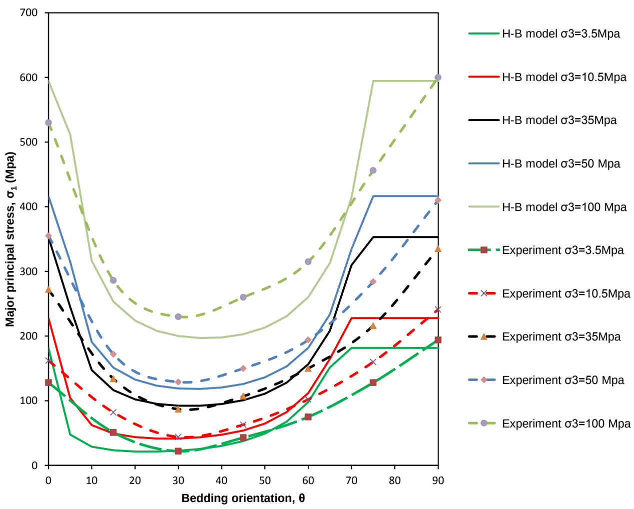



*Figure 4 Major principal stress vs bedding oreintation of Martinsberg slate from experimental data as compared to Hoek - Brown Model(H-B)*

### 2.3 Tien and Kuo's criteria for transversely isotropic rocks

Tien and Kuo's failure criterion is one of the criteria that describes the anisotropic response of transversely isotropic rocks such as: schist, slates, genesis, shale, sand stone shale and phylites, where the properties of these rocks are highly dependent on the direction of schistocity.

This model is based on Jaeger's, 1960 criteria and maximum axial strain theory. Unlike Hoek-Brown model, Tien and Kuo, 2001 based their criterion on the deviatoric stress causing the failure of transversely isotropic.

$$S_{1(\theta)} = \sigma_{1(\theta)} - \sigma_3 = \frac{2(c_i+\sigma_3 tan\phi_i)}{(1-tan\phi_i tan\theta)sin2\theta} \quad (9)$$

In addition to the classical modes of failure for transversely isotropic rocks, failure in rock matrix structure and failure along weak plane, Tien and Kuo, 2006 have demonstrated on experimental investigation of simulated transversely isotropic rock that such rocks also may fail due to axial strain accumulation. The axial strains can be calculated from the constitutive equation of transversely isotropic materials, equation (10).

$$\begin{bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{33}\\ \gamma_{23}\\ \gamma_{31}\\ \gamma_{12}\end{bmatrix} = \begin{bmatrix}\frac{1}{E} & \frac{-\upsilon'}{E} & \frac{-\upsilon}{E'} & 0 & 0 & 0\\ \frac{-\upsilon'}{E} & \frac{1}{E} & \frac{-\upsilon}{E'} & 0 & 0 & 0\\ \frac{-\upsilon}{E'} & \frac{-\upsilon'}{E} & \frac{1}{E} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{G'} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{1}{G'} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G}\end{bmatrix} \begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \tau_{23}\\ \tau_{31}\\ \tau_{12}\end{bmatrix} \quad (10)$$

Where $E, E', \nu, \nu', G$ and $G'$ are elastic constants of a transversely isotropic material on plane of isotropy and on a plane normal to it.

As it is apparetnt, constitutive equation is usually given in local coordinate system but if we have to deal with axial strains we need to transform the elastic parameters of the material in to global system.

$$[K] = [Q]^T[K'][Q] \quad (11)$$

Where $[K]$ and $[K']$ are stress-strain relationship matrices in global and local coordinate axes respectively. $[Q]$ is a suitable 6X6 matrix involving direction cosines of local axes in the global axes.

In this paper a transformation technique introduced especially for transversely isotropic rocks by Amadei, 1996 has been adopted . Amadei's proposal uses the theory of elasticity and considering the above conditions for anisotropic and assuming uniform distribution of stress and strain in the specimen, $\varepsilon_x, \varepsilon_y, \varepsilon_z$ and $\gamma_{xy}$ can be related to the applied stress $\sigma$ in uniaxial compression test as follows:

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 & a_{16} \\ a_{12} & a_{22} & a_{23} & 0 & 0 & a_{26} \\ a_{13} & a_{23} & a_{33} & 0 & 0 & a_{36} \\ 0 & 0 & 0 & a_{44} & a_{45} & 0 \\ 0 & 0 & 0 & a_{45} & a_{55} & 0 \\ a_{16} & a_{26} & a_{36} & 0 & 0 & a_{66} \end{bmatrix} \begin{bmatrix} 0 \\ S_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (11)$$

Where $a_{11}, a_{22}, \ldots, a_{66}$ are transformation matrix constants and $S_1$ deviatoric stress$(\sigma_1 - \sigma_3)$.

$$a_{12} = -\frac{\nu'}{E'}\sin^4\theta - \frac{\nu'}{E'}\cos^4\theta + \frac{\sin^2 2\theta}{4}\left(\frac{1}{E'} + \frac{1}{E} - \frac{1}{G'}\right)$$

$$a_{22} = \frac{\sin^4\theta}{E} + \frac{\cos^4\theta}{E'} + \frac{\sin^2 2\theta}{4}\left(\frac{1}{G'} - \frac{2\nu'}{E'}\right)$$

$$a_{23} = -\frac{\nu'}{E'}\cos^2\theta - \frac{\nu}{E}\sin^2\theta$$

$$a_{26} = \sin^2 2\theta\left[\cos^2\theta\left(\frac{1}{E'} + \frac{\nu'}{E'}\right) - \sin^2\theta\left(\frac{1}{E} + \frac{\nu'}{E'}\right)\right] - \frac{\sin 2\theta\cos 2\theta}{2G'}$$

Since the experimental data from Donath F.A., 1964, *Table 1,* is used to verify Tien and Kuo's model, only the stress-strain relationship in the major prinicipal direction, corresponding to a uni-aixl test condition, is considered.

$$\varepsilon_{yy} = a_{22}S_1 \quad (13)$$

$$E_y = \frac{1}{a_{22}} \quad (14)$$

Failure of transversely isotropic rocks may occur due to strain accumulation i.e., when the maximum failure strain reaches prior to rock matrix or weak plane failure. Each bedding plane orientation has its own maximum failure strain which can be expressed in terms of the maximum deviatoric stress.

$$S_{1(\theta)} = \frac{\varepsilon_{yf}}{a_{22}} = E_y\varepsilon_{yf} \quad (15)$$

Where: $\varepsilon_{yf}$ is the maximum failure strain that varies with confining pressure independent of bedding plane orientation.

According to Tien and Kuo's model, the strength of a transversely isotropic at bedding orientation angles of θ=0 and θ=90 can be described using Hoek and brown criteria, equation (4). However, several test data is required to calibrate the model parameters where failure along the weak plane (bedding plane) is expected. The experimental data on Martinsburg slate tested by Donath F.A., 1964, has once more been used to check the criterion proposed by Tien and Kuo, 2001.

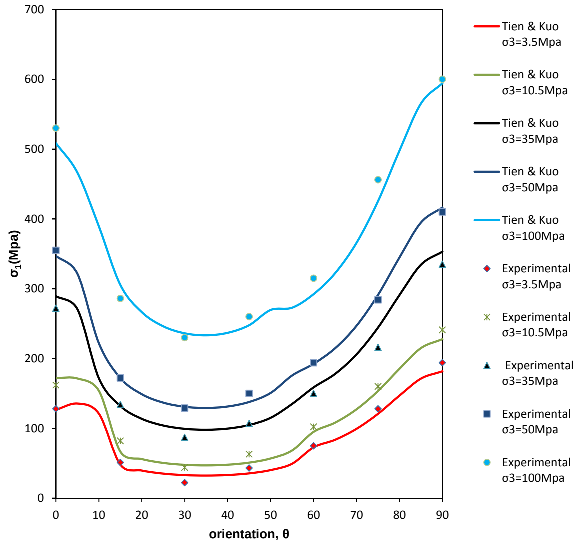



*Figure 5 Comparison of experimental data (after Donath F.A., 1964) and Tien and Kuo's 2001 model*

The comparison between the experimental work and the prediction from the model shows good agreement for different confinig pressures. In addition, we can observe the sharp change on the curvatures of the model at points where the the assumed failure is changing from non-sliding to a sliding type of failure. This is simply due to change in the mathematical expression used for the criteria at this specific points. Comparing this criteria with the other discontinuous type of criteria discussed above Tien and Kuo's criterion has only seven parameters that can be determined from a few triaxial tests . Moreover, It is versatile and fairly accurate.

## 3 FEM SIMULATION OF TRANSVERSELY ISOTROPIC ROCK

There have been significant advances in the use of the computational methods in rock mechanics in the last three decades. The complexities associated in the discipline of rock mechanics necessitates the use of modern numerical methods. With the rapid advancements in computer technology, numerical methods provide extremely powerful tools for analysis and design of engineering systems with complex factors that was not possible or very difficult with the use of the conventional methods, often based on closed form analytical solutions.

Rock mass is largely discontinuous and anisotropic by nature, and this makes rock a difficult material to represent it mathematically for numerical modeling. However, several FEM based tools have been developed to capture the essential mechanical behaviour of anisotropic geo-materials. In this paper a 2D FEM program, PLAXIS, is used to simulate the mechanical (stress-strain) relationships of a trasnasversely isotropic material.

### *3.1 Use of Interface in layered rocks to simulate transversely isotropic rocks*

Anisotropic rocks, which have different mechanical properties in different directions, require a lot of experimental samples to determine its properties. Because of high variability of the natural rock due to their formation process, geological environment, weathering and mineral composition, it is difficult to obtain a great number of field specimens with uniform properties. Therefore it is necessary to use artificially created rocks.

Artificially prepared rocks can be used to simulate an interstratified rock blocks that is transversely isotropic rocks. It is enough to use a bilaminated artificial rock in order to simulate variation of strength along with the inclination of the inter bedding planes between the two constituent materials. Here, artificially layered rock triaxial data has been extracted from literature, Yi-shao etal, 1999. Each of the layers are isotropic rocks which can be described with Mohr-Coulomb criterion. For demonstrating purpose artificial rock made of two different cement types is used to represent a layered transversely isotropic rock. The experimental results are shown below:

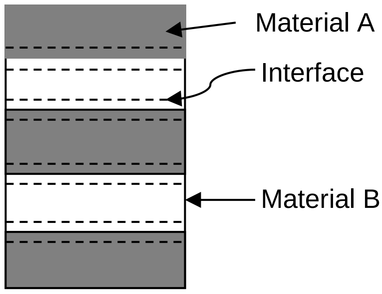



*Figure 6 A stack of two different materials representing Bi-layered rock*

The objective is to simulate the real situations mentioned above in PLAXIS 2D. The layered rock is modeled as two different isotropic rocks stalking on one another. The contact between the two materials has been simulated using interface elements.

For demonstrating purpose artificial rock made of two different cement types is used to represent a layered transversely isotropic rock. The experimental results are shown below:

*Table 2 Maximum principal stress on artificially prepared rock (After Yi-Shao etal, 1999)*

| **$\sigma_3$ [Mpa]** | **$\sigma_1$ [Mpa]** | | | | | | |
|---|---|---|---|---|---|---|---|
| | **θ=0** | **θ=15** | **θ=15** | **θ=30** | **θ=45** | **θ=60** | **θ=90** |
| 0 | 14.3 | 12.7 | 12.6 | 4.44 | 3.57 | 6.72 | 21.4 |
| 2.5 | 21.3 | 20.5 | 17.0 | 10.77 | 10.4 | 13.02 | 26.5 |
| 5 | 25.4 | 24.2 | 22.1 | 20.5 | 18.8 | 20.1 | 33.1 |
| 10 | 29.3 | 29.3 | 27.1 | 25.0 | 26.9 | 30.5 | 44 |

From the aforementioned triaxial experiment on the artificially prepared layered material, it was observed that the following kinds of failures were common:

__*Overall failure mode:*__ this failure mode can be obtained when all the constituent layers reach their ultimate strength or the critical conditions. It can due to the fact that the constituent layers are ductile and the strength difference between the layers is small.

*Mid inclination failure mode:* this will happen when only the weakest layer fails, i.e., when $\tau_w = c_w + \sigma_w \tan\phi_w$ is maintained (Where $\tau_w$ and $\sigma_w$ are shear and normal stress in the weak plane; $c_w$ and $\phi_w$ are the cohesion and friction angle of the weak layer.

*Interface failure mode*: such a failure happens along the contact (interface) area between the two layers. In this case the failure criteria is dependent on the strength of the interface not on the constituent layers, i.e., $\tau_i = c_i + \sigma_i \tan\phi_i$ (Where $\tau_i$ and $\sigma_i$ are shear and normal stress on the interface; $c_i$ and $\phi_i$ are the cohesion and friction angle of the weak layer.

*Low inclination failure mode:* for most artificially prepared layered rocks with bedding angle orientations of $0 \le \theta \le 60$ (low inclination), failures occurs in single consistent layer. In other words low inclination failures are dominated by single layer.

From experimental tests on the two constituent cement like materials of the bi-layered rock used by Yi-shao etal,1999, the following strength and stiffness parameters were found. The interface parameters were obtained by back calculating experimental data for failure along the contact (interface) area. For this case it was observed that failure along the interfaces happens when $\theta = 60$.

*Table 3 Stiffness and strength parameters of the constituent materials*

| | c [Mpa] | ϕ [°] | E [Mpa] | ν [-] |
|---|---|---|---|---|
| Material A | 10.2 | 25 | 7000 | 0.22 |
| Material B | 6.34 | 15 | 2500 | 0.13 |
| Interface | 0.98 | 29 | - | - |

Using the above back calculated parameters, a traixial test was simulated. The geometrical model consists of two different materials, each following Mohr-Coulomb material model. These materials are assumed to be layered on each other and the contact area between the materials is modeled using interface element. For the general geometry of the sample a medium mesh was used but at the interfaces a refined mesh was used.

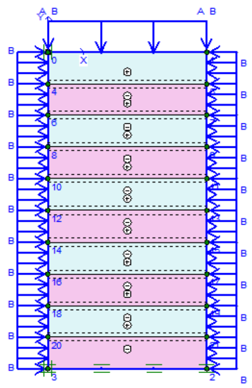


*Figure 7 Geometrical model of the layerd material in Plaxis – simulating a triaxial test*

The results from the FEM simulations have shown that when the plane containing the interface is oriented in such a way that $45 \le \theta \le 75$ the strength decreases and attains it minimum value at $\theta = 60$. This observation was also manifested from the experimental results. Figure shows the comparison between experimental data and numerical simulation, which are in a very good agreement.

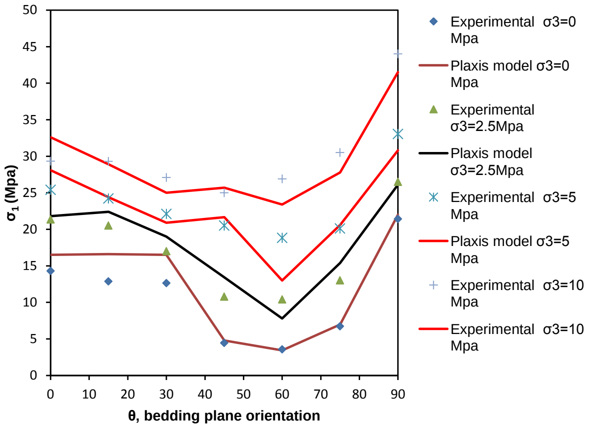



*Figure 8 Comparison of results from experiment (Yi-shao etal, 1999) and Plaxis*

## 4 CONCLUSION AND RECOMMENDATION

Summaries of the different failure criteria and numerical simulation using PLAXIS have been discussed in the previous sections. Herein the important findings are presented.

- The compressive strength behavior of anisotropic rocks is a function of both the confining pressure and the orientation of the bedding plane to the applied stress. The minimum compressive strength usually occurs within $30 \le \theta \le 60$, whereas the maximum strength occurs at $\theta = 90$.
- Anisotropic rocks with weak planes of weakness exhibit two kinds of failures which are failure along weak plane and failure in the rock matrix.
- Failure criteria based on weakness plane can describe the strength behavior of anisotropic rocks. However, it is difficult to implement in Finite element programs.
- Layered rock simulation in PLAXIS can give good approximation to transversely isotropic rocks.

From discussions in this paper some questions are answered relative to the strength behavior of anisotropic rocks, but those questions left unanswered will be evident that a great deal more work remains to be done in this field. A better understanding of the mechanics of jointed rock mass behavior is a problem of major significance in geotechnical engineering, and it is an understanding to which both the traditional disciplines of soil mechanics and rock mechanics can and must contribute. The author hopes that the ideas presented will contribute towards this understanding and development.

## 5 REFERENCES


Amadei B. (1996), Importance of anisotropy when estimating and measuring in situ stresses in rock, International journal of rock mechanics and mining science, Vol. 33, No. 3, pp. 293 – 325.

Casagrande, A. & Carillo, N. (1944), Shear failure of Anisotropic Material, Proceedings of Boston society of civil engineers, Vol. 31, pp. 74 – 87.

Donath F.A. (1964), A strength variation and deformational behaviour of anisotropic rocks, State of stress in Eath's crust, New York, Elsevier, pp. 281-297

Duvea, G. etal (1998), Assessment of failure criteria for strongly anisotropic geomaterials, Mechanics of cohesive frictional materials, Vol. 3, pp. 1-26.

Hoek E. (1983), Strength of jointed rock masses,, Geotechnique, Vol. 33, No. 3, pp. 187 – 223.

Hoek E. & Brown E.T., (1980), Empirical strength criterion for rock masses, Journal of geotechnical engineering division, ASCE, 106, No. GT9, pp. 1013 – 1035.

Jaeger, J.C. (1971), Friction of rocks and stability of rock slope, Geotechnique, Vol. 21, No. 2, pp. 97 – 134.

Jaeger, J.C. & Cook, N.G. (1969), Fundamentals of rock mechanics, Chapman & Hall, London.

Jaeger, J.C. (1960), Shear failure of anisotropic rocks, Geological Magazine, Vol. 97, No. 1, pp. 65 – 72.

Pariseau, W.G. (1972), Plasticity theory for anisotropic rocks and soils, Proceedings of 10th symposium on Rock Mechanics, Vol. 1, AIME, pp. 267-295.

Tien, Y.M. & Kuo, M.C. (2001), A failure criterion for transversely isotropic rocks, International journal of rock mechanics and mining science, Vol. 38, pp. 399 – 412.

Tien, Y.M. etal. (2006), An experimental investigation of the failure mechanism of simulated transverse isotropic rocks, International journal of rock mechanics and mining science, Vol. 43, pp. 1163 – 1181.

Yi-Shao lai, etal. (1999), Modified Mohr-Coulomb type micro mechanical failure criteria for layered rocks, International journal for numerical and analytical methods in geomechnics, Vol. 23, pp. 451 – 460.