Load bearing capacity of railway embankments

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ABSTRACT

When designing embankments that carry significantly strip load e.g. railway embankments, the Limit Equilibrium Method (LEM) "Method of Slices" with circular slip surfaces has been widely used for decades. With today's software and computationally capabilities more realistic slip surfaces than the circle can easily be analyzed by Method of Slices e.g. through methods where the circle is optimized to a non-circular slip surface or by Finite Element Methods (FEM). In Eurocode 7 it is stated that circular slip surfaces normally are appropriate in homogenous and isotropic soils, but as this article will show, circular slip surfaces might lead to a significant overestimation of the bearing capacity for embankments carrying significant load. The bearing capacity for such embankments will be analyzed and compared using J.Brinch Hansen closed form solution, Limit Equilibrium Method and Finite Element Method. The analyses will be carried out for homogenous and isotropic soils, as well as for multilayered conditions in drained and undrained conditions. A clear recommendation is formulated.

Keywords: Slope stability, Optimized slip surface, LEM, FEM, bearing capacity

1 MOTIVATION

Slope stability has for decades been evaluated using Limit Equilibrium Methods (LEM) and circular formed slip surfaces. Even though thousands of circular slip surfaces today can be calculated within seconds, this does not guarantee that the lowest factor of safety is found for that simple reason, that the most adverse slip surface in most soils is non-circular. This article will demonstrate that using circular slip surfaces in LEM calculations might lead to significant overestimation of the bearing capacity of embankments carrying significant strip load i.e. railway embankments, even in homogeneous and isotropic soils.

2 BACKGROUND

In the Method of Slices, the potential sliding mass is discretized into vertical slices. This was first done in Gothenburg in Sweden in 1916 and presented by Petterson (1955). Fellenius (1936), Janbu (1954) and Bishop (1955) further developed the method during the mid-1950s. During the 1960s the development of computers and the work of Morgenstern and Price (1965) and Spencer (1967) led to more rigorous formulations and effective iterations that took into account the interslice forces (normal and shear) and that would solve for force equilibrium as well as moment equilibrium (Krahn, 2004). Over the last decade or so Finite Element analysis (FEM) in geotechnics has expanded enormously and today - as this method is becoming more and more common – the differences to LEM is truly starting to emerge.

3 FACTOR OF SAFTEY

The Method of Slices' factor of safety FoS is in the more rigorous formulations by e.g. Morgenstern & Prices determined as the ratio between driving and resisting forces summed up over the entire slip surface leading to a global mean FoS. These methods full-fill force as well as moment static equilibrium through an iterative procedure "determining" the necessary interslice forces to comply with just that. This means that the slice forces may not be realistic locally, but the global factor of safety in nonetheless realistic as the integration procedure involved smoothens out local irregularities (Kahn, 2004). LEM analysis does not full-fill the strain/stress relations ship within the mass as FEM does. In FEM calculations the factor of safety is calculated through a strength reduction method (SRM) or “phi – c reduction” which lowers the strength of the soil (φ’ and c’) until a clear failure mechanism is formed. Both methods calculates the factor of safety as the ratio between the total available shear strength along the slip surface divided by the summation of the gravitational driving forces (mobilized shear) (Kahn, 2004), and the factor of safety is thereby analogue to the partial safety factors for soil strength used in Eurocode 7.

4 METHODOLOGY

In order to investigate the slope stability for slopes carrying significant strip loads such as railway embankments, the ability for the embankment to carry ultimate limit state load will be investigated as follows. The topic will be divided in three scenarios.

1. Embankment of infinite height
2. Embankment of finite height
3. Embankment on soft soil

The bearing capacity is analysed using design values of soil strength in LEM, thus f = 1.0 gives the design value of the bearing capacity. In FEM characteristic values of soil strength is used, meaning that a factor of safety M_{SF} equal to the desired partial factor of soil strength gives the design value of the bearing capacity (also see paragraph 4.2). The reason for this is, that FEM can have numerical difficulties near failure (M_{SF} =1).

4.1 Geometry

The basic geometry chosen for the local bearing capacity analysis will resemble a typical cross section of a railway embankment use by the Danish Railway authorises (Banedanmark).
The load is evenly distributed to the soil over a width of 2.5m. The distance from the load centre line to the embankment crest is 3.8m and the slope angle is 21.8° (1:2.5 slope). The vertical distance from the slope surface to a virtual line origin at the corner of the load area and running parallel to the slope is 1.02m (this will be used when introducing the overburden in the analytical analysis).

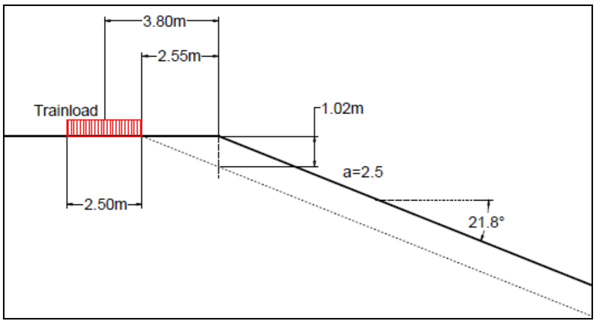

Figure 1a. Typical cross section.

For the embankment of finite height a height of 5 m is chosen, and for the embankment on soft soil a stabilizing berm is included as shown on figure 1b.

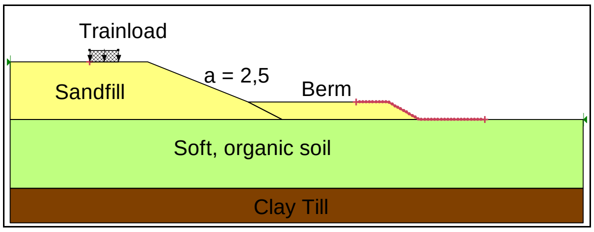

Figure 1b. Model. Embankment on soft soil

4.2 Soil and water conditions

The characteristic soil parameters considered are:

Table 1 Soil parameters. Drained

	γ [kN/m³]	**φ’$_k$ [°]**	**c’$_k$ [kPa]**
Sandfill	20	38	0
Clayfill	20	30	5
Clay	20	30	5

Table 2. Soil parameters. Undrained

	γ [kN/m³]	**c$_{u,k}$ [kPa]**
Clayfill	20	80
Soft soil	13	30
Clay	21	100

The stiffness parameters for the FEM modelling are not important for this ULS analysis, and therefore not presented here. The phreatic line is assumed of no influence to the problems analysed.

For the analytical analysis and for the LEM analysis, design values of the above strength parameters are used utilizing partial safety factors on soil strength, γ_M, according to the Danish National Annex to Eurocode 7: DS/EN 1997 – DK NA:2013 which for soil friction angle and effective cohesion is $\gamma_\varphi = \gamma_{c'} = 1.32$ and for undrained cohesion is $\gamma_{cu} = 1.98$ (values given for structures in consequence class 3 e.g. railways). For the FEM analysis the characteristic strength parameters given in table 1 and 2 are used leading to the target factor of safety $M_{sf} = 1.32$ and 1.98 respectively. The FEM bearing capacity is calculated with varying angle of dilatancy equal to $\psi = 0$, $\psi \approx \varphi - 30° = 8°$ and $\psi = \varphi = 30°$ in the sand fill material. For drained analysis in clay fill $\psi = 0$ is adopted. When undrained soil (clay or soft soil) and sand are combined in a FEM model, the joint target factor of safety is $M_{SF} = 1.32$, as the clay undrained strength is given as $c_{u,model} = (c_{u,k} / 1.98) \cdot 1{,}32$.

4.3 Methods

For the analytical bearing capacity the Brinch Hansen solution is adopted (Brinch Hansen, 1970), (reproduced by Hansen, B. (1978)). The LEM analysis is performed using GeoStudio 2007 Slope/w and Morgenstern & Price's Method. For Finite Element analyses Plaxis 2D 2015 is used. In LEM the bearing capacity is found by trial and error using design values of soil strength demanding f = 1.0. In FEM analysis the bearing capacity is found through a "Safety analysis" loading the embankment by trial and error until $M_{SF} = 1.32$ thus giving the desired design strength values and design bearing capacity. 15-node elements and plane strain (2D) conditions are assumed in all FEM analyses.

5 EMBANKMENT OF INIFINTE HEIGTH

5.1 Brinch Hansen solution

The Brinch Hansen (JBH) analytical solution yields the following design bearing capacities for a 2.5 m wide foundation placed at the crest of a 1:2.5 slope with an overburden pressure of $q' = 1.02\text{m} \cdot 20\text{kN/m}^3 = 20.4\text{kPa}$ as indicated on figure 1.

Table 3. Brinch Hansen bearing capacities

	Drained [kPa]	**Undrained [kPa]**
Sand	314	-
Clay	188	177

5.2 Sand embankment

By trial and error in LEM, the load intensity giving FOS = 1.00 is found yielding the load bearing capacity (R_d/A) for the model investigated. For the sand embankment the circular slip surface (CSS) gives $R_d/A = 507$ kPa, cf. figure 2.
For the same model the optimized (non-circular) slip surface (OSS) (using default optimizations settings) gives $R_d/A = 305$kPa, cf. figure 3.
When modelling the exact same geometry in finite element with FEM the geometry of the failure mechanism is shown in figure 4a.
With a load of $R_d/A = 348$ kPa FEM gives af $M_{SF} = 1.32$ ($\psi = 8°$) as required for the fully developed failure mechanism.

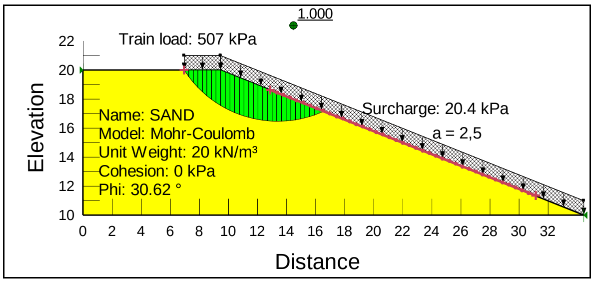

Figure 2. Slope/w. Sandfill. (CSS)

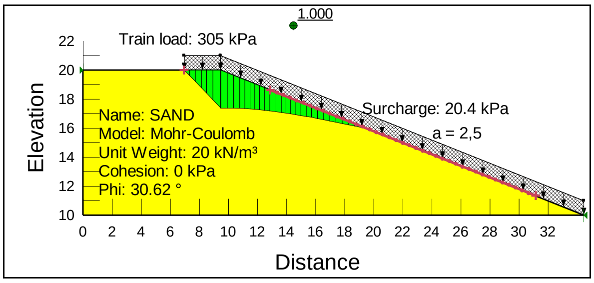

Figure 3. Slope/w. Sandfill (OSS)

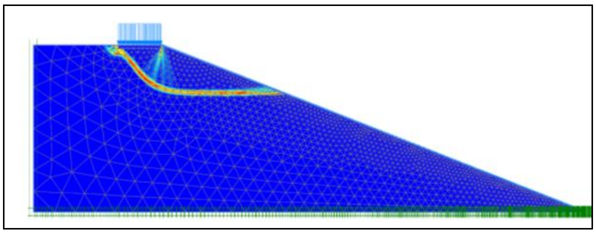

Figure 4a. FEM. Sandfill. Deviatoric strain. ψ = 8 °

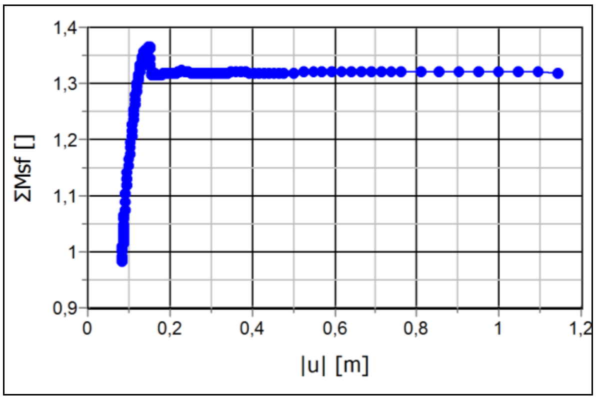

Figure 4b. FEM. Sandfill. M_{SF} vs. deformation

For this purely frictional case the follwing dependenci of the angle of dilatency ψ is found:

Table 4. FEM. Dependenci of ψ

	ψ = 0	ψ = 8°	ψ = 38°
Bearing capacity [kPa]	300	348	425

5.3 *Clay embankment – Drained*

For the same model using effective strength parameters for clay (drained conditions), LEM gives R_d/A=240 kPa using circular slip surface (CSS):

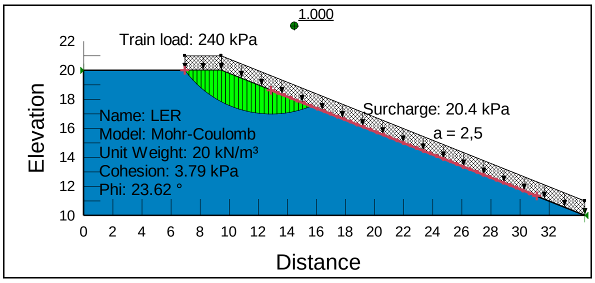

Figure 5. LEM CSS. Clay fill. Drained

The LEM optimized non-circular calculation (OSS) gives R_d/A = 179kPa:

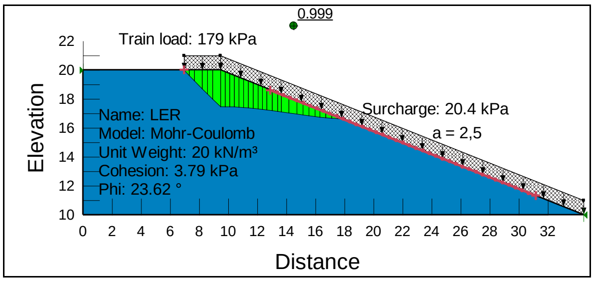

Figure 6. LEM OSS. Clay fill. Drained

For the exact same geometry FEM gives a collapse load of R_d/A = 200 kPa.

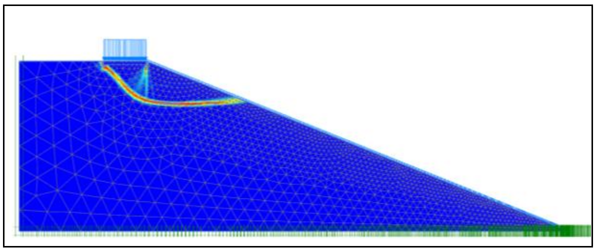

Figure 7a. FEM. Clay fill. Drained. Deviatoric strain

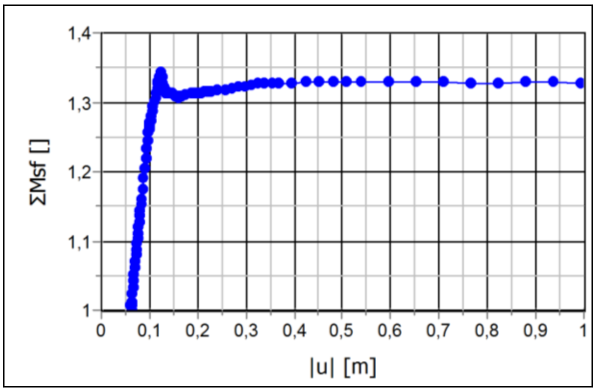

Figure 7b. FEM. Clay fill. Drained

5.4 *Clay embankment –Undrained*

LEM CSS yields a bearing capacity of R_d/A =182kPa (figure 8), and the LEM OSS R_d/A =168 kPa (figure 9).

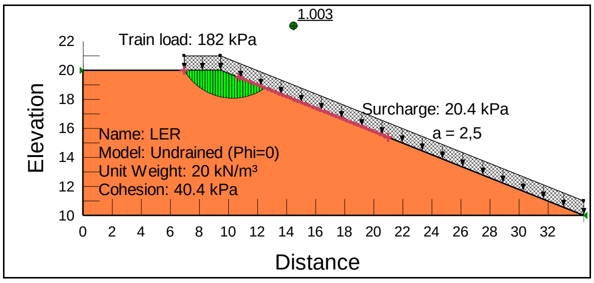

Figure 8. LEM CSS. Clay fill. Undrained

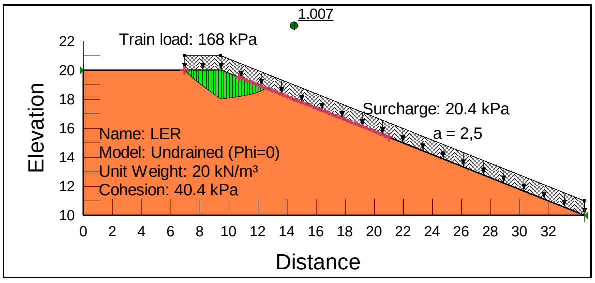

Figure 9. LEM OSS. Clayfill. Undrained

In the undrained FEM analysis a minor adjustment to the strength profile has to be introduced in order analyze the bearing capacity. If constant undrained shear strength is used, the failure mechanism tends to extent to the model boundary leading to total stability failure mechanism instead of a bearing capacity (which is investigated here). Therefore the undrained shear strength profile is modified to c_u = 78kPa at the surface with an increase with depth of Δc_u = 1kPa/m giving an approximate average of c_u = 80 kPa within the depth of interest. The bearing capacity is found to be R_d/A' = 190 kPa (target is M_{SF} = 1.98)

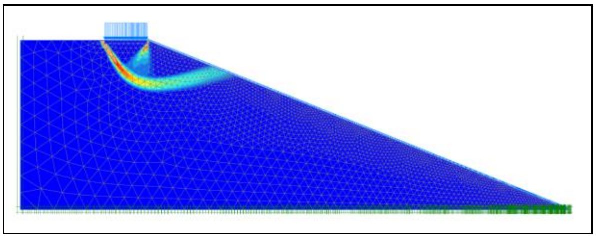

Figure 10a. FEM. Clay fill.Undrained. Deviatoric strain

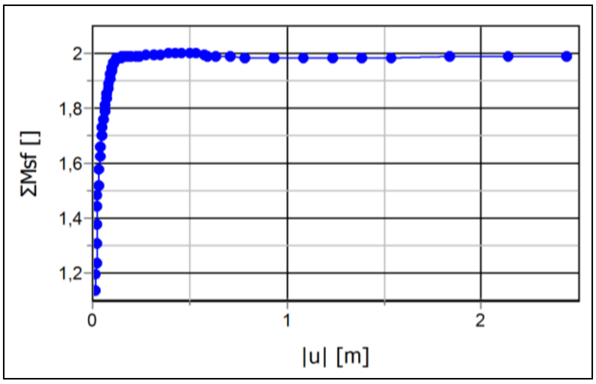

Figure 10b. FEM. Clay fill.Undrained. M_{SF} vs deformation

5.5 Summation

The results above can be summarized as shown in table 5.

Table 5. Results. Embankment of infinite height

Sand fill	JBH	LEM		
		CSS		OSS
	[kPa]	[kPa]		[kPa]
	314	507		305
Deviation	0%	+61%		-3%
		FEM		
		[kPa]		
		ψ = 0	ψ = 8°	ψ = 38°
		300	348	425
Deviation		-4%	+11%	+35%

Clay fill Drained	JBH	LEM	
		CSS	OSS
	[kPa]	[kPa]	[kPa]
	188	240	179
Deviation	0%	+28%	-5%
		FEM	
		[kPa]	
		200	
Deviation		+6%	

Clay fill Undrained	JBH	LEM	
		CSS	OSS
	[kPa]	[kPa]	[kPa]
	177	182	168
Deviation	0%	+3%	-5%
		FEM	
		[kPa]	
		190	
Deviation		+7%	

6 EMBANKMENT OF FINITE HEIGTH

For this situation no analytical solution exists, so this scenario is analysed by LEM and FEM only.

6.1 Sandfill embankment on clay (drained)

For a sandfill embankment placed on clay, the drained bearing capacity is R_d/A = 489 kPa using the circular slip surface (CSS):

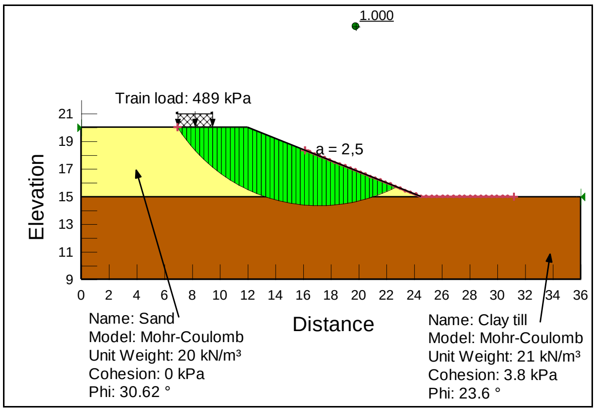

Figure 11. LEM CSS. Sandfill on clay. Drained

The optimized slip surface shows a bearing capacity of R_d/A = 304 kPa as shown on figure 12.

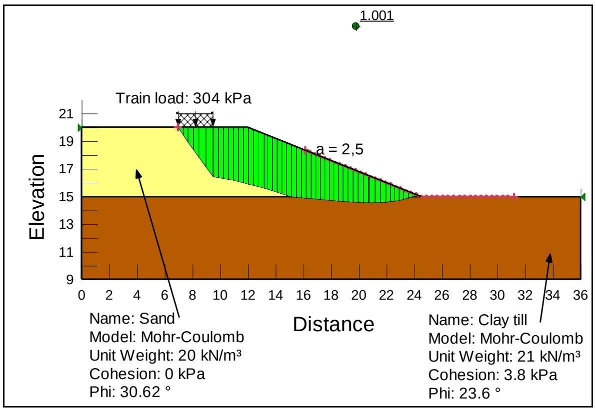

Figure 12. LEM OSS. Sandfill on clay. Drained

FEM gives a bearing capacity R_d/A = 295 / 328 kPa (ψ = 0° / 8° in sand fill).

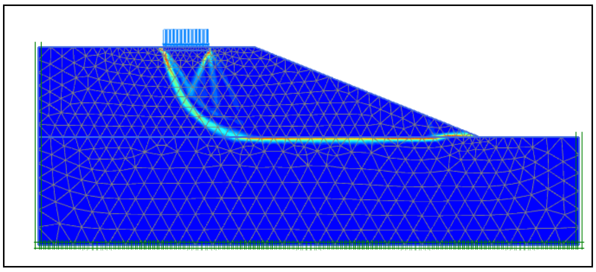

Figure 13. FEM. Sandfill on clay. Drained. Deviatoric strain. ψ = 8° in sand fill

6.2 *Sandfill embankment on clay(undrained)*

For a sand fill embankment placed on clay the undrained bearing capacity is R_d/A = 514 kPa using the circular slip surface (CSS), cf. figure 14.

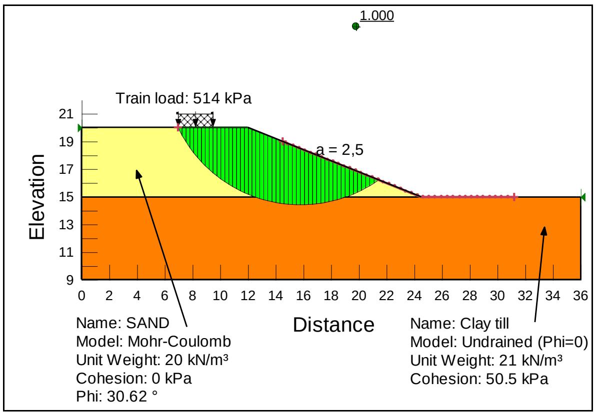

Figure 14. LEM CSS. Sandfill on clay. Undrained

The optimized slip surface shows a bearing capacity R_d/A = 289 kPa, cf. figure 15.

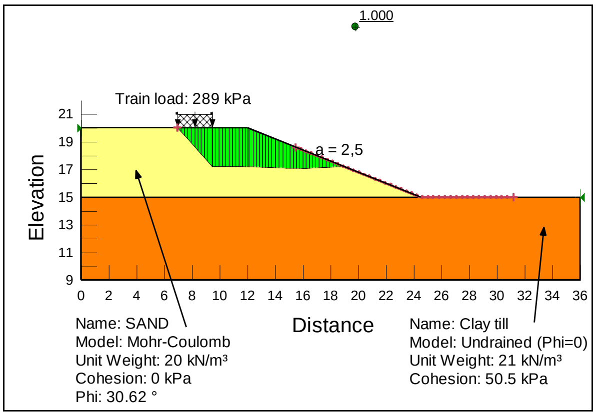

Figure 15. LEM OSS. Sandfill on clay. Undrained

FEM yields a capacity of R_d/A = 290 / 340 kPa (ψ = 0° / 8° in sand fill):

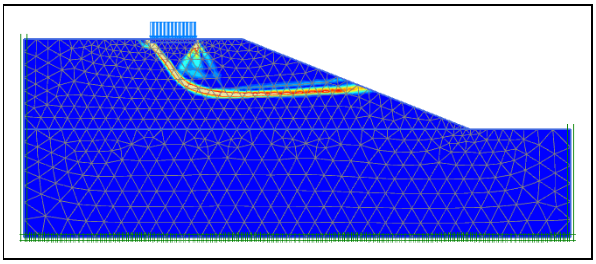

Figure 16. FEM. Sandfill on clay. Undrained. Deviatoric strain. ψ = 8° in sand fill

Below the LEM CSS and LEM OSS capacities are compared to the FEM analysis

Table 6. Results. Embankment of finite heigth

	LEM		**FEM**[1]
	CSS	**OSS**	
	[kPa]	**[kPa]**	**[kPa]**
Drained	**489**	**304**	**295**[1] **328**[2]
Deviation	**+49%**	**+3%** **-7%**	**-**
Undrained clay	**514**	**289**	**290**[1] **340**[2]
	[kPa]	**[kPa]**	**[kPa]**
Deviation	**+51%**	**0%**[1] **-15%**[2]	**-**

[1] ψ = 0° in sand fill. [2] ψ = 8° in sand fill

Note the similarity between LEM OSS and FEM slip surfaces, and that neither penetrates the clay layer as LEM CSS does.

If the model shown in figure 14 is loaded with the ultimate load from LEM OSS and FEM (with ψ = 0°) i.e. 290 kPa, the slip surface changes radically and a FoS of 1.198 is found, cf. figure 14a:

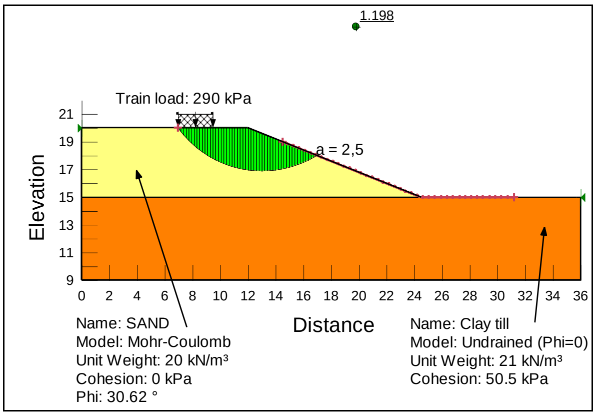

Figure 14a. Model cf. figure 14 loaded with 290 kPa yields FoS = 1.198 with LEM CSS.

On the other hand, if the embankment is loaded with ultimate load from LEM CSS i.e. 514 kPa FoS drops to 0.839 when using LEM OSS, cf. figure 14b:

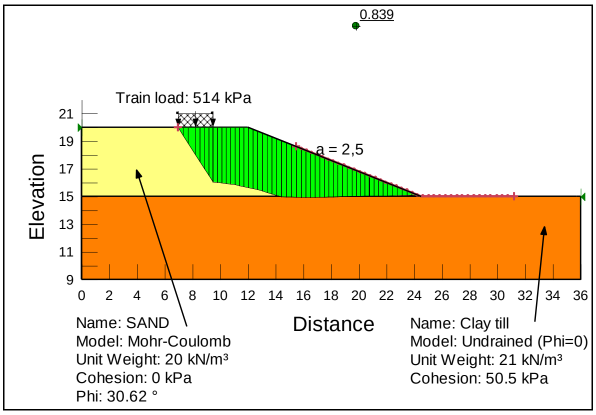

Figure 14b. Model cf. figure 14 loaded with 514 kPa yields FoS = 0.839 with LEM OSS.

7 EMBANKMENT ON SOFT SOIL

When stabilizing an old embankment, a berm could be placed at the embankment foot as countermeasure, as shown on figure 1b. For this situation no analytic solution exists, and therefore the analysis must be conducted by means of LEM or FEM calculations. The LEM CSS yields a bearing capacity of $R_d/A = 167$ kPa, cf. figure 17, whereas the LEM OSS yields a capacity of $R_d/A = 35$ kPa, cf. figure 18, with undrained conditions in the soft soil.

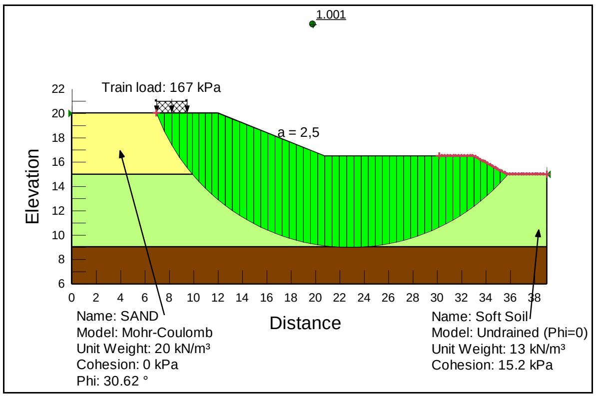

Figure 17. Embankment on soft soil. LEM CSS.

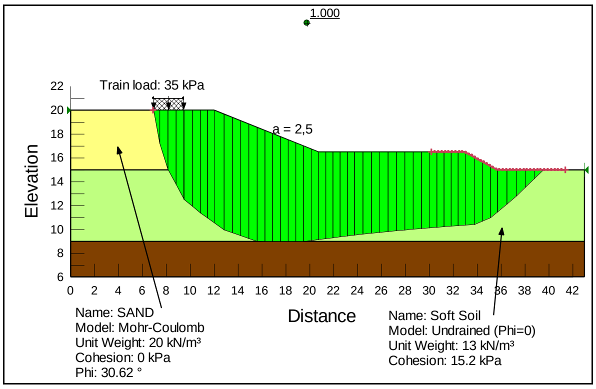

Figure 18. Embankment on soft soil. LEM OSS.

FEM yields the following slip surface and bearing capacities for various ψ of the sand fill material. As in the previous scenarios the bearing capacities are found through a safety analysis in FEM with a target factor of safety $M_{SF} = 1.32$ giving the desired design values of the soil strength. As the partial factor for undrained strength is 1.98, the soft soil in FEM is modelled with an undrained strength $c_{u,model} = (c_{u,k} / 1.98) \cdot 1{,}32$ so a common target factor of safety $M_{SF} = 1.32$ can be used.

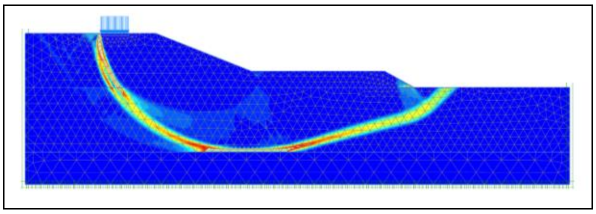

Figure 19. Embankment on soft soil. FEM. Deviatoric strain.

Note the similarity between LEM OSS and FEM slip surfaces.

Table 7. Dependency of ψ

	ψ = 0	**ψ = 8°**	**ψ = 38°**
Bearing capacity [kPa]	55	59	62

No significant dependency of ψ is found in this case.
Below the LEM CSS and LEM OSS capacities are compared to the FEM analysis:

Table 8. Results. Embankment on soft soil

	LEM		FEM
	CSS	**OSS**	
Soft soil Undrained	**167**	**35**	**59**
Deviation	**+183%**	**-41%**	**-**

8 CONCLUSIONS

For the embankment of infinite height an analytical solution exists and when comparing the results from Limit Equilibrium Methods (LEM) and Finite Element Methods (FEM) with this, it is very clear that LEM with optimized slip surface and FEM both gives results very close to the analytical solution (- 5% to +11%), whereas LEM with circular slip surface over predicts the bearing capacity significantly in the drained calculations for both sand fill and clay fill embankments (+61% and +28%). In the undrained calculations all methods gives results close to the analytical solution (≈+5%).
The over prediction with circular slip surface (LEM CSS) in the sand fill embankment of infinite height is equal to a reduction in the partial factor of safety from $\gamma_\varphi = 1.32$ to $\gamma_\varphi = 1.16$ and in the similar clay fill embankment from $\gamma_\varphi = \gamma_c = 1.32$ to $\gamma_\varphi = \gamma_c = 1.2$. This indicates how much the LEM CSS results are on the unsafe side compared to FEM.
For the 2 layer embankment of finite height an analytical solution does not exist. If the LEM results are compared to FEM, it is again clear that the circular slip surface from LEM significantly over predicts the bearing capacity (+50%). LEM with optimized slip surface yields results very close to FEM in the drained analysis (+3% to -7% depending on ψ used in FEM), and a bit more in the undrained analysis (-18%).
Finally the analyses for the embankment on soft soil with a berm at the embankment foot shows that the circular slip surface very significantly over predicts the capacity compare to FEM (+168%), while the LEM optimized slip surface significantly under estimate the bearing capacity when compared to FEM (-41%).
If the FEM model for the embankment on soft soil with c_u = 30 kPa should be able to carry a load of 167 kPa as found in the LEM CSS analysis, the partial factor of safety γ_{cu} =1.98 must be reduced to γ_{cu} =1.27. This indicates how much the LEM CSS result is on the unsafe side. Similar if FEM only should carry the LEM OSS load of 35 kPa, the factor of safety γ_{cu} =1.98 could be raised to γ_{cu} =2.15, which indicates how much LEM OSS is on the safe side compared to FEM for this model.

From the above it seems clear that the LEM with circular slip surface (LEM CSS) in general significantly over predicts the bearing capacity of an embankment. The over prediction found in the examples presented here is so significant, that use of circular slip surfaces should be avoided when evaluating the stability of slopes – at least when a load is present at the embankment top. If used anyway, the safety of the embankment risks being far from what was intended. The optimized slip surface (LEM OSS) in general predicts bearing capacities very comparable to FEM.
Eurocode 7 states as principal text that:

> *2.4.1.6(P): Any calculation model shall be either accurate or err on the side of safety*

Further Eurocode 7 states that:

> *11.5.1(5) Where ground or embankment material is relatively homogeneus and isotropic, circular slip surfaces should normally be assumed.*

It is found from the work presented here, that LEM with circular slip surfaces does not meet 2.4.1.6(P), and that 11.5.1(5) should be seriously questioned - at least when significant loads are present at the embankment top.

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