Modeling soft Scandinavian clay behavior using the asymptotic state

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ABSTRACT

The deformation of soft or sensitive Scandinavian clays can be a complex process in which the structure is continuously changing to accommodate applied loading. This paper presents a pragmatic way of describing this process using differential equations for the Cauchy stress tensor where the normal consolidated state and asymptotic behavior play an important role. An advantage to this approach is that for simple strain paths (e.g. undrained triaxial and oedometer tests) the development of stress can be found analytically. The model provides stress-strain relationships in principle for any strain path, but small strain behavior where isotropic unloading is involved requires further research.

Keywords: soft clays, asymptotic states, barodesy

1 INTRODUCTION

Many of the soft clay models used today have been developed within the theory of plasticity. This framework allows constitutive models to be defined by the elastic stiffness and a minimum of two scalar functions (Hill 1950), the plastic potential and the yield function, where the first will give the orientation of the plastic strain increments and the second will provide the boundary between elastic and elastic-plastic behavior for any stress state.

The well-known Cam-Clay (CC) model (Schofield and Wroth 1968) and subsequent variations have the critical state as a fundamental basis, which is a state observed during large shear deformations when there is no further change in effective stresses or volume (Wood 1990). The CC model was originally developed with reconstituted clays in mind, i.e. rather simple clays without the bonds and fabric (Burland 1990, Länsivaara 1999) found in natural clays. Later much advancement has been made to include the effects of anisotropy, destructuration, rate-dependence and creep (Dafalias 1986, Wheeler, Näätänen et al. 2003, Dafalias, Manzari et al. 2006, Grimstad and Degago 2010, Grimstad, Degago et al. 2010, Olsson 2013).

Although this framework elegantly provides complex stress-strain relationships in a continuum of three spatial dimensions, advanced models may have complicated hardening rules which will simultaneously influence the behavior. This can make model calibration challenging because hardening parameters can influence both isotropic and deviatoric deformation behavior and may or may not have a direct physical meaning. In particular, providing direct input of an undrained ADP shear strength profile (Grimstad, Andresen et al. 2012) in effective stress based models can be a challenge with one set of parameters.

This paper presents a pragmatic model for soft Scandinavian clays formulated as a system of differential equations. It is based on the concept of asymptotic / steady states

(Schofield and Wroth 1968, Poulos 1971) and has three basic underlying hypotheses. Although originally developed separately, the modeling ideas resembles well those of Barodesy (Kolymbas 2012) and have also some similarities to the works of (Joseph 2010) and (Mašín 2012). The model presented here provides stress-strain relationships in principle for any strain path, but with the main focus being on deformation behavior not involving isotropic unloading as it is considered a different process from consolidation. A pragmatic way of modeling isotropic unloading has been described but does require further research for small strain cycles.

Some very useful features from Barodesy are already incorporated and the future objective is to fully formulate the model within this framework.

1.1 Definitions

The formulation presented herein applies to soft or sensitive clays starting from an initial condition towards well defined asymptotic states, which are reached through a continuous deformation process; any reversal is the start of a new process. The strain increment $d\varepsilon_{ij}$ is considered to be acting on the soil while the Cauchy stress tensor σ_{ij} is the reaction or response. Although this is a choice of definition, it is a sensible one considering that shear stress may exhibit post-peak softening making the strain non-unique for a given stress level. On the other hand, strain will always correspond to a unique stress level for the process described here. The isotropic and deviatoric components of the strain increment are defined as:

$$d\varepsilon_p = \frac{d\varepsilon_{kk}}{3} \quad (1)$$

$$d\varepsilon_q = \sqrt{\frac{2}{3}\left(d\varepsilon_{ij} - d\varepsilon_p\delta_{ij}\right)\left(d\varepsilon_{ji} - d\varepsilon_p\delta_{ji}\right)} \quad (2)$$

where $d\varepsilon_p$ is positive in compression and $\delta_{ij} = \delta_{ji}$ is the Kronecker delta. This implies that for triaxial loading with ε_a for axial strain and ε_r for radial strain we have:

$$d\varepsilon_q = \frac{2}{3}\left(d\varepsilon_a - d\varepsilon_r\right) \quad (3)$$

The Cauchy stress tensor is assumed to be composed as follows

$$\sigma_{ij} = p\left(\eta_{ij} + \delta_{ij}\right) \quad (4)$$

All stresses are effective and thus the commonly used '-notation has been omitted for convenience. The component p has the unit of stress but is a scalar; η_{ij} is without units but does contain components. In this paper η_{ij} is said to represent the orientation of the stress tensor and p represents the equivalent isotropic magnitude and is the mean effective stress, i.e.

$$p = \frac{\sigma_{kk}}{3} \quad (5)$$

where p is positive in compression. The split of a stress into p and η_{ij} allows the stress measures to be seen as separate: alone their formulation can be quite simple while the combination of the two may provide rather complex soil behavior. For example, the deviatoric stress $s_{ij} = \eta_{ij} p$ may exhibit a peak even though η_{ij} and p does not; if a soft clay reaches its asymptotic orientation before the magnitude of stress a peak will typically develop. The scalar equivalent of η_{ij} is here called the invariant η and is defined as

$$\eta = \sqrt{\frac{3}{2}\eta_{ij}\eta_{ji}} \quad (6)$$

The deviatoric (shear) stress q can then be found as η times p. The more general η_{ij} has been used throughout the derivations so that information about the spatial components of stress is retained.

2 HYPOTHESES

The model has three underlying hypotheses which are stated on the basis of natural soft Scandinavian clay behavior observed mainly in laboratory tests.

The three hypotheses are:

1. Applying strain increments to a clayey soil will asymptotically make it normal consolidated.
2. The stress will eventually reach an asymptotic orientation which is uniquely defined by the orientation of the strain increment responsible for bringing it into this state.
3. The magnitude of the asymptotic stress is uniquely related to the void ratio of the soil.

The three hypotheses above are similar to those which form the basis of Barodesy. In particular, the first two hypotheses are more or less identical to Goldscheider's first and second rule (Goldscheider 1967) but rephrased slightly to accommodate the behavior of clays and the normal-consolidated (NC) state. It follows from the second hypothesis that the asymptotic orientation of stress is independent of initial conditions and the path taken. The third hypothesis is important because it defines the asymptotic value of hydrostatic stress as a state parameter unique to the current void ratio.

From the rules of calculus it follows that contributions from the strain increment components on the stress state can be incrementally superimposed. It is assumed that this is also true in a physical sense.

3 CONSTITUTIVE EQUATIONS

A measure of the process of becoming normal-consolidated has traditionally been given to the over-consolidation ratio, OCR:

$$OCR = \frac{p^m}{p} \tag{7}$$

Where p is the magnitude of current stress and p^m is the stress which retains memory ("m") of prior loading. The incremental form of eq. (7) defines the main differential equation for p and is given by:

$$dp = p\left(\frac{dp^m}{p^m} - \frac{dOCR}{OCR}\right) \tag{8}$$

If the development of the NC state dp^m and the process of becoming normal-consolidated $dOCR$ is given as a function of strain increments, then the development of dp is fully defined. In the derivations below an approach is taken to separate the relative complex stress-strain behaviour into simpler parts: First the process of deformation is seen as an interaction between the normal- and overconsolidated state, then a separation of the magnitude and orientation of stress is made, and finally the contribution from the isotropic and deviatoric strain components are incrementally superimposed. In total this makes the resultant stress behaviour relatively complex while maintaining simplicity in the individual expressions. It is also assumed that memory (OCR) is only related to the magnitude of stress and not the orientation. The change in the stress orientation is thus given as:

$$d\eta_{ij} = \frac{\partial \eta_{ij}}{\partial \varepsilon_p} d\varepsilon_p + \frac{\partial \eta_{ij}}{\partial \varepsilon_q} d\varepsilon_q \tag{9}$$

The following formulations are suggested:

$$\frac{\partial \eta_{ij}}{\partial \varepsilon_q} = \left(\eta_{ij,\infty} - \eta_{ij}\right) \cdot k_{\eta q} \tag{10}$$

$$\frac{\partial \eta_{ij}}{\partial \varepsilon_p} = \frac{\partial \eta_{ij}}{\partial \varepsilon_q} \frac{k_{\eta p}}{k_{\eta q}} \qquad (11)$$

where $k_{\eta p}$ and $k_{\eta q}$ are proportionality constants and $\eta_{ij,\infty}$ is the asymptotic value of η_{ij}. The equations (9) to (11) can be integrated analytically for simple strain paths. The ratio κ is introduced:

$$\kappa = \frac{d\varepsilon_q}{d\varepsilon_p} \qquad (12)$$

For a constant κ ratio the analytical expression is found as:

$$\frac{\eta_{ij}}{\eta_{ij,\infty}} = 1 - \left(1 - \frac{\eta_{ij,0}}{\eta_{ij,\infty}}\right) e^{-(k_{\eta p} + k_{\eta q}\kappa)\varepsilon_p} \qquad (13)$$

where $\eta_{ij,0}$ is the initial orientation and 'ij' denotes each individual component of the tensor. The asymptotic value $\eta_{ij,\infty}$ must be related to the orientation of the strain increment. The framework of Barodesy elegantly provides this relation for the components of the stress tensor through an exponential mapping function (Medicus, Kolymbas et al. 2015) . For the purpose used in this paper it provides the internal friction angle φ and the coefficient at rest K_0^{NC} as possible calibration parameters. For pure deviatoric deformation the steady state strength follows more or less the Matsuoka-Nakai failure criterion (Fellin and Ostermann 2013) while in an oedometer condition it will represent a K_0^{NC} condition. For pure isotropic deformation the asymptotic orientation is zero.

Due to space limitations the presentation here has been made brief, but reference is made to the works of (Kolymbas 2012, Medicus, Fellin et al. 2012) for important derivations and a thorough description.

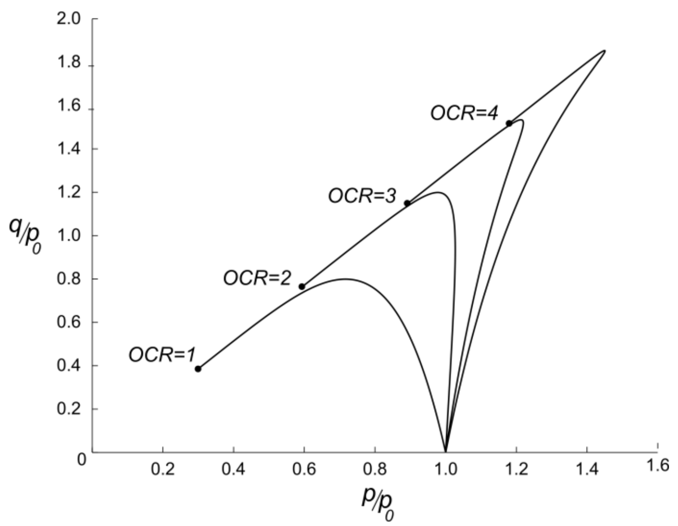

Figure 1. Simulation of an undrained triaxial compression test for various OCRs.

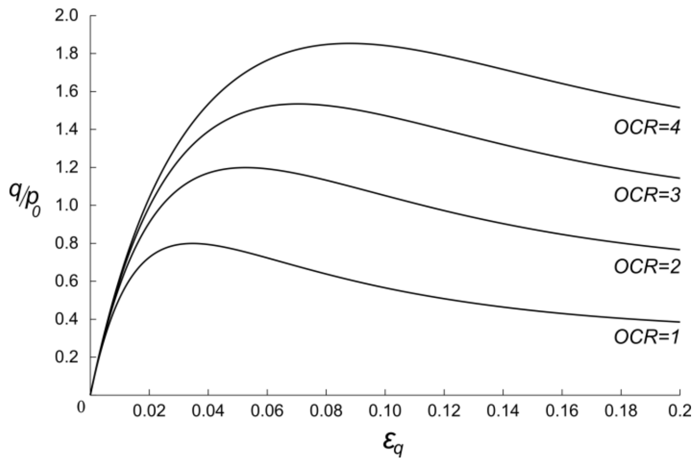

Figure 2. Development of undrained shear strength for various OCRs.

3.1 Normal-consolidated state

Following the simplification in the previous section, i.e. memory only being related to the magnitude of stress, the stress invariant p^m will describe the soil as if it were normal-consolidated and will go towards a steady state value p_∞^m during deviatoric deformation but will grow "exponentially" for isotropic dominated deformation. As such it is a continuously updated target for p .

The change in p^m is given as:

$$dp^m = \frac{\partial p^m}{\partial \varepsilon_p} d\varepsilon_p + \frac{\partial p^m}{\partial \varepsilon_q} d\varepsilon_q \quad (14)$$

The following formulations are suggested:

$$\frac{\partial p^m}{\partial \varepsilon_p} = p^m \cdot k_{mp} \quad (15)$$

$$\frac{\partial p^m}{\partial \varepsilon_q} = \left(p_\infty^m - p^m \right) \cdot k_{mq} \quad (16.a)$$

$$\frac{dp_\infty^m}{d\varepsilon_p} = p_\infty^m \cdot k_{mp,\infty} \quad (16.b)$$

The proportionality constants k_{mp}, k_{mq} and $k_{mp,\infty}$ will later be related to physical quantities and must be non-dimensional. The equations above are written down on the basis of observed behavior: the first equation describes the exponential stress-strain curve seen in isotropic dominant deformation; the second is the steady state stress due to deviatoric (undrained) conditions and the third equation follows from hypothesis number three.

In the derivations to follow the proportionality constant of eq. (16.b) has been set equal to that of eq. (15) but need not be in general; whenever they are equal the "critical state line" (CSL) and the "normal-consolidation line" (NCL) will be parallel with slope k_{mp} in a $\varepsilon_p - \ln p$ diagram.

It should be noted that as long as dp^m divided by p^m is non-dimensional then all stress-strain curves can be normalized by the initial magnitude of stress (fig. 1 and 2) for a given OCR. This can be seen from eq. (8). By considering κ constant for a monotonic strain path, the analytical expressions are obtained as:

$$\frac{p^m}{p_{\infty,0}^m} = e^{k_{mp}\varepsilon_p} \left(1 - \left(1 - \frac{p_0^m}{p_{\infty,0}^m} \right) e^{-\kappa k_{mq}\varepsilon_p} \right) \quad (17)$$

$$\frac{p_\infty^m}{p_{\infty,0}^m} = e^{k_{mp}\varepsilon_p} \quad (18)$$

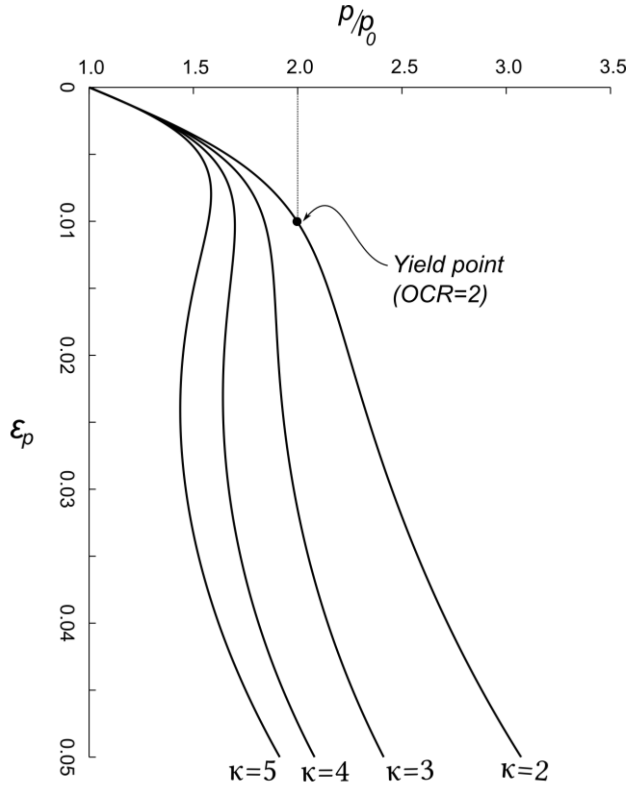

Figure 3. Effect of increasing κ, i.e. introducing more shear deformation relative to $\kappa = 2$ (oedometer condition)

Where p_0^m and $p_{\infty,0}^m$ are the initial values of p^m and p_∞^m respectively. The term outside the parenthesis of eq. (17) describes the behavior expected to be seen for pure isotropic strain. The main term inside the parenthesis is due to the deviatoric strain component and will decay and eventually disappear. This is important because as long as there is continuous (positive) isotropic deformation the soil will not fail even though it may temporarily appear to do so, but eventually show an exponentially increasing strength. An illustrative plot of this can be seen in figure 3. Dividing eq. (17) by eq. (18) gives:

$$\frac{p^m}{p_\infty^m} = 1 - \left(1 - \frac{p_0^m}{p_{\infty,0}^m} \right) e^{-\kappa k_{mq}\varepsilon_p} \quad (19)$$

It can be seen that for pure isotropic deformation ($\kappa = 0$) the "contractancy" p^m divided by p_∞^m remains constant throughout the strain path, but goes towards unity whenever deviatoric deformation is involved.

This is important because it implies a continuous structural degradation with shear strain. First consider a soft clay specimen being consolidated along a K_0- line in a triaxial cell to its in-situ stress. If sheared in an undrained manner the shear strength will typically exhibit a peak due to its relative high contractancy, even if the soil is normal-consolidated. Next compare an "identical" sample consolidated along the same line but far beyond any prior loading so that eq. (19) becomes close to unity. Even though the stress has steadily been increasing due to the isotropic component of deformation the deviatoric strain has been degrading the structure. If sheared undrained the second sample would not exhibit any peak even if the first sample did. This can be seen from the incremental form of the deviatoric stress tensor:

$$ds_{ij} = d\eta_{ij} \cdot p^m + dp^m \cdot \eta_{ij} \tag{20}$$

At this "residual" state, i.e. p^m equal to p_∞^m, only the first term involving the orientation of stress contributes to the development of shear strength since p^m remains constant. Because the orientation of stress does not exhibit a peak in this formulation the shear stress does not either for this case.

3.2 Over-consolidated state

This section describes the continuous process of the soil becoming normal consolidated, with emphasize made on the word continuous because mathematically the process never ends but goes asymptotically towards its steady state value. Any deformation, except where isotropic unloading is involved, will make the soil more normal consolidated and make OCR go towards unity:

$$OCR_\infty = 1 \tag{21}$$

In a most pragmatic form the development of OCR with strain can be formulated as:

$$dOCR = \frac{\partial OCR}{\partial \varepsilon_p} d\varepsilon_p + \frac{\partial OCR}{\partial \varepsilon_q} d\varepsilon_q \tag{22}$$

$$\frac{\partial OCR}{\partial \varepsilon_q} = (1 - OCR) \cdot k_{oq} \tag{23}$$

$$\frac{\partial OCR}{\partial \varepsilon_p} = \frac{\partial OCR}{\partial \varepsilon_q} \cdot \frac{k_{op}}{k_{oq}} \tag{24}$$

Where k_{op} and k_{oq} are proportionality constants. For a constant κ ratio the analytical expression is given as:

$$OCR = 1 - (1 - OCR_0) e^{-(k_{op} + \kappa k_{oq})\varepsilon_p} \tag{25}$$

Where OCR_0 is the initial over-consolidation ratio which can typically be found from an oedometer test using standard determination procedures related to e.g. a measure of curvature. There are some exceptions to this which will be discussed later.

3.3 Isotropic unloading

The only component of deformation that will not make the soil become more normal-consolidated is isotropic unloading and as such is considered a separate process. In this case OCR can be said to be created as the memory of prior loading is being retained. A pragmatic way of incorporating isotropic unloading is to replace eq. (24) with the following expression:

$$\frac{\partial OCR}{\partial \varepsilon_p} = -OCR \cdot k_{op} \tag{26}$$

Since $d\varepsilon_p$ is negative for isotropic unloading the equation above will make OCR go exponentially towards infinity, i.e. making p go towards zero. In addition eq. (11) must change sign. The resulting behavior is realistic for larger isotropic strain cycles but is inaccurate for small cycles such that further research is needed.

3.4 Analytical solution

In the previous section the analytical solutions for a simple κ strain path was presented. The expression for p^m of eq. (17) can be combined with the definition of OCR eq. (7) to give the following expression for the magnitude of stress:

$$\frac{p\left(\kappa,\varepsilon_p\right)}{p^m_{\infty,0}}=\frac{e^{k_{mp}\varepsilon_p}\left(1-\left(1-\frac{p^m_0}{p^m_{\infty,0}}\right)e^{-\kappa k_{mq}\varepsilon_p}\right)}{1-\left(1-OCR_0\right)e^{-\left(k_{op}+\kappa k_{oq}\right)\varepsilon_p}} \quad (27)$$

The expression for the individual components of stress orientation was found as:

$$\frac{\eta_{ij}\left(\kappa,\varepsilon_p\right)}{\eta_{ij,\infty}}=1-\left(1-\frac{\eta_{ij,0}}{\eta_{ij,\infty}}\right)e^{-(k_{\eta p}+k_{\eta q}\kappa)\varepsilon_p} \quad (28)$$

Inserting the above equations into eq. (4) will provide the components of the Cauchy stress tensor. In case of a two component ($q-p$) formulation the deviatoric stress is found as η multiplied by p where η is found from eq. (6). The second argument ε_p could be replaced by the deviatoric strain ε_q divided by κ if desired and then let $\kappa\rightarrow\infty$ for undrained conditions. Besides simulating simple monotonic strain paths directly eq. (27) and (28) can be used to calibrate the 5 proportionality constants to quantities found in laboratory tests.

4 PARAMETER DETERMINATION

From the governing equations it seems natural that initial inclinations in stress-stress and stress-strain plots are used to calibrate the proportionality constants. There seems though to be a relative flexibility in choosing the input parameters as long as they come from simple strain-driven laboratory tests. In this paper the following is used:

Stress orientations:

1. The friction angle, φ
2. The earth pressure coefficient, K_0^{NC}

For the NC condition:

3. The oedometer modulus number, m
4. The "contractancy", p^m_∞ / p^m_0
5. The undrained shear strength, s_u^{NC} for a triaxial compression test.

For a given OCR:

6. The undrained shear strength, s_u^{OC} for a triaxial compression test.
7. The initial shear stiffness, G_0.
8. The initial oedometer stiffness, M_0.

The proportionality constants can often be obtained directly if related to initial inclinations, but whenever the undrained shear strength is involved a system of non-linear equations must be solved at the initial stage. In the implementation the requirements above are given as 6 equations with expressions obtained from eq. (27) and (28). After these equations have been solved the model is initialized and the proportionality constants are given values. If the initial OCR is different than that calibrated for, the formulation will predict the behavior based on internal relations. All stress-strain curves can be normalized by the initial stress. While the undrained shear strength is considered the most important for typical short term ultimate limit state cases, the same procedure can in principle be done for other responses considered important.

In the next section the conditional equations for the undrained shear strength is provided for the normal-consolidated condition in a two-component ($q-p$) formulation.

4.1 Deviatoric deformation

During undrained shearing the contribution from the isotropic strain increment is considered negligible due to the relative incompressibility of the pore water. Introducing simplifying factors:

$$\alpha=\frac{k_{mq}}{k_{\eta q}},\ \beta_0=\frac{\eta_0}{\eta_\infty}$$

$$\beta=\frac{\eta_p}{\eta_\infty},\ \gamma=\frac{p^m_0}{p^m_\infty}-1 \quad (29)$$

where 'subscript p' now denotes peak. Consider a soil specimen that has been pre-consolidated beyond its in-situ stress so that it has become normal-consolidated. For an undrained laboratory test taken to shear failure the deviatoric (peak) strength is found as:

$$\frac{q_p}{q_\infty} = \beta \left\{ 1 + \gamma \cdot \left(\frac{\beta - 1}{\beta_0 - 1} \right)^{\alpha} \right\} \quad (30.a)$$

$$\frac{\alpha}{(1-\beta)} - \frac{1}{\beta} \left(1 + \frac{1}{\gamma} \cdot \left(\frac{\beta - 1}{\beta_0 - 1} \right)^{-\alpha} \right) = 0 \quad (30.b)$$

In the case of triaxial tests q_p is equal to $2 \cdot s_u$ where s_u is the undrained shear strength. The two equations (30) are non-linear and coupled and can be used to prescribe the undrained shear strength within certain limits, i.e.:

$$\frac{q_p}{p_0} \le \eta_\infty \le \frac{q_p}{p_\infty} \quad (31)$$

If the peak value is larger than the maximum value then the soil must be over-consolidated, while the minimum value will be the steady state value found at large shear strains. As seen from the two equations above (and β) the value of hydrostatic stress at peak p_p will be determined by the model formulation whenever the undrained shear strength is given as input. After the two eq. (30) is solved for α and β, the proportionality constant $k_{\eta q}$ can be obtained as:

$$k_{\eta q} = \frac{1}{\varepsilon_{q,p}} \ln\left(\frac{1-\beta_0}{1-\beta} \right) \quad (32)$$

Where $\varepsilon_{q,p}$ is the deviatoric strain at the peak. It can be seen that the ratio α calibrates the shape in a $q-p$ plot while the value of $k_{\eta q}$ is used to calibrate the stiffness in a $q-\varepsilon_q$ plot. For the OC state a similar procedure can be performed but it is not shown here. As discussed earlier the initialization of proportionality constants are done numerically.

4.2 Yield point

For an oedometer test the ratio κ between deviatoric and isotropic strains is equal to 2 while for a purely isotropic deformation it is zero. Due to the relative high κ value the influence of shear strains on the stress state can become significant at the beginning of an oedometer test, in particular for very sensitive clays. First consider that the model has an initial mean stress of p_0. An oedometer test is simulated and the yield point is defined as the following stress point:

$$p_y = p_0 \cdot OCR_0 \quad (33)$$

where 'y' denotes yield. If p_y is found from an oedometer curve and p_0 is the in-situ stress level, the initial over-consolidation ratio OCR_0 can be obtained from the equation above. The yield point is typically found where it is expected, fig. 4 and 5, but has not been found to be related to a specific measure of curvature. In simulated tests where either the soil is very sensitive or the test boundaries induce much deviatoric deformation, i.e. high κ values, the soil will show heavy stiffness degradation, fig. 3, and standard definitions of the yield point likely no longer applies.

5 IMPLEMENTATION

The governing equations are given in incremental form and will have to be solved for the 9 unknowns:

$$v = \begin{Bmatrix} p \\ \eta_{ij} \\ p^m \\ p_\infty^m \end{Bmatrix} \quad (34)$$

After the vector v has been found at each increment the Cauchy stress tensor can be obtained from eq. (4). The 9 governing equations are assembled in a residual vector:

$$r = \begin{Bmatrix} \Delta p - p\left(\frac{\Delta p^m}{p^m} - \frac{\Delta OCR}{OCR}\right) \\ \Delta\eta_{ij} - \frac{\partial\eta_{ij}}{\partial\varepsilon_p}\Delta\varepsilon_p - \frac{\partial\eta_{ij}}{\partial\varepsilon_q}\Delta\varepsilon_q \\ \Delta p^m - \frac{\partial p^m}{\partial\varepsilon_p}\Delta\varepsilon_p - \frac{\partial p^m}{\partial\varepsilon_q}\Delta\varepsilon_q \\ \Delta p^m_\infty - \frac{dp^m_\infty}{d\varepsilon_p}\Delta\varepsilon_p \end{Bmatrix} \quad (35)$$

Here the operator d has been replaced by Δ to denote that the infinitesimal differentials have been replaced by finite difference approximations. Inserted for the constitutive expressions, the residual vector r is now a set of equations which should become close to zero, a matter which is solved with the Newton-Raphson method.

The implementation is generally stable but does require small steps so that which should have been infinitesimal does not become too finite, in particular since the differential equations are “exponential” in nature and errors could accumulate.

6 RESULTS

Various test cases have been simulated to show the different features described in this paper. The input parameters were given as follows:

Table 1: Example soil properties

Friction angle φ	32°
Coefficient at rest K_0^{NC}	0.55
Oedometer modulus m	25
Contractancy p^m_∞ / p^m_0	0.25
Undrained shear strength, s_u^{NC}	$0.4\,p_0$
For an OCR of 2.0:	
Undrained shear strength, s_u^{OC}	$0.6\,p_0$
Shear stiffness, G_0	$75\,p_0$
Oedometer stiffness, M_0	$150\,p_0$

Table 2: Proportionality constants (obtained)

k_{mq}	k_{mp}	$k_{\eta q} = k_{\eta p}$	k_{op}	k_{oq}
13.73	25.00	58.21	240.15	25.52

The figures 1-5 show various responses for this set of input parameters.

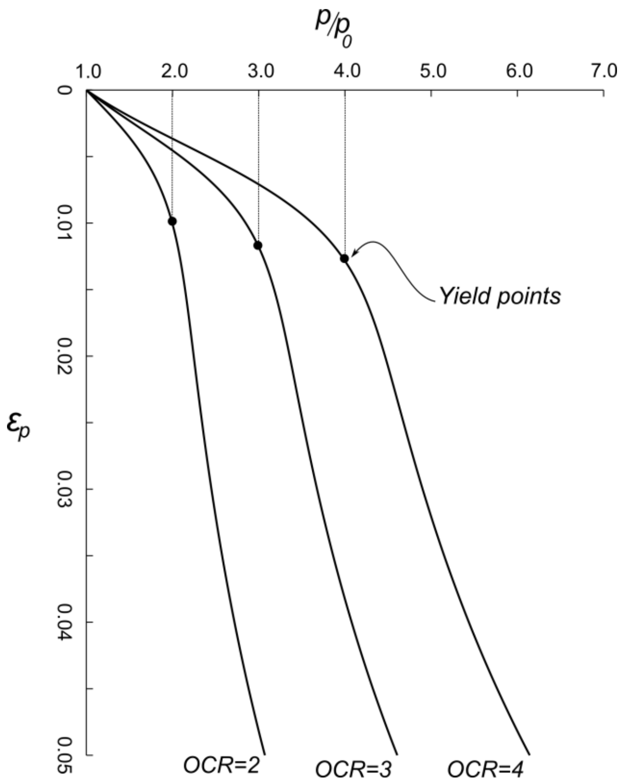

Figure 4. Simulation of an oedometer test for various OCRs.

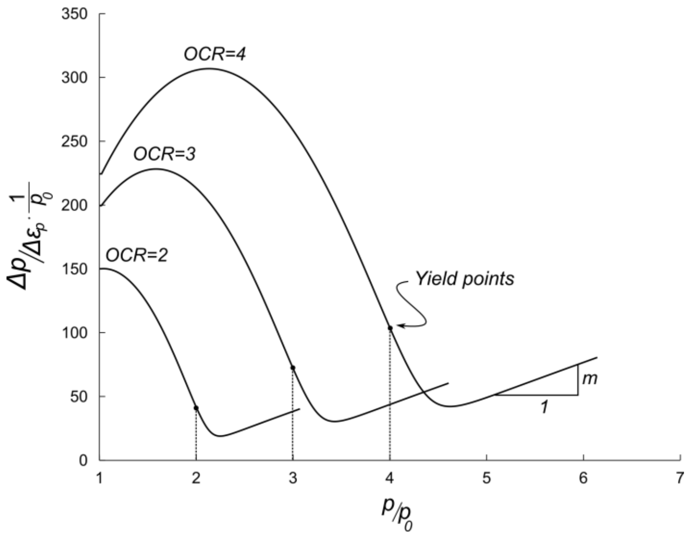

Figure 5. Another representation of the simulated oedometer test.

7 CONCLUSIONS

A constitutive model for soft Scandinavian clays has been formulated as a system of differential equations in a pragmatic way. The model provides stress-strain relationships in principle for any strain path, but deformation where isotropic unloading is involved requires further research for small strain cycles. The formulation is only a minor contribution but does show that it is possible to predict realistic soil behavior from simple assumptions on the process of deformation and the use of calculus. The framework Barodesy takes a similar approach and the future objective is to incorporate the model fully within this framework. Future work will also focus on providing the undrained ADP shear strength ratios as input.

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