# A rate-dependent creep model for anisotropic soft soils

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ABSTRACT
*A rate-dependent anisotropic constitutive model for natural soft clays is proposed. The effect of stress-induced plastic anisotropy is investigated assuming kinematic, i.e., rotational, material hardening. The rate-dependent behaviour of soil is incorporated by adopting an elasto-viscoplastic framework. Predictions of the new model are compared with experimental data, obtained from triaxial testing, for a particular clay soil. These predictions are also compared with independent predictions of other soil models appearing in the literature.*

**Keywords:** Creep, viscoplasticity, constitutive modelling.

## 1 INTRODUCTION

It is well known that many natural soft soils exhibit rate dependence in their response to mechanical loading. This rate dependence may manifest in the short term as soil straining that depends on the rate of application of stress, or in the longer term as continuing creep under constant effective stress. In addition, such soils often also display anisotropic straining and even destructuring in responding to these stress changes. These aspects of soil behaviour are particularly relevant for soft soils initially consolidated in the field under one-dimensional (or $K_o$) conditions and then subsequently loaded in shear due to the application of construction loads. Predicting such behaviour has been the subject of a number of previous studies, e.g., Kutter and Sathialingham (1992), Rowe and Hirchberger (1998), Kelln et al. (2008), Yin et al. (2010), Sivasithamparam et al. (2015).

In this paper the problem of predicting the rate-dependent response of an anisotropic material is addressed. A new, relatively simple constitutive model for soft soil is proposed based on the well established theory of elasto-viscoplasticity. The validity of the proposed model is demonstrated by presenting model predictions for undrained triaxial compression stress paths and different rates of loading, and then comparing some of these predictions with published experimental data for a particular clay soil and previously published predictions of other soil models.

## 2 VISCOPLASTIC FORMULATION

### 2.1 Basic Assumptions

The elasto-viscoplastic soil model described in the following is essentially an extension of the viscoplastic formulation presented by Kelln et al. (2008). Their original model has been augmented here by allowing the possibility of an anisotropic response of the soil.

For this model the following assumptions are adopted:

- Both elastic and time-dependent viscoplastic strains may occur in the soil;
- The elastic strain components are fully recoverable;
- The viscoplastic stain rates and components are given by the so-called "overstress theory" first proposed by Perzyna (1963, 1966) and later adopted for soil by Kutter and Sathialingham (1992), Yin et al. (2010) and others, i.e., they can be defined in terms of gradients of a viscoplastic potential function; and
- The viscoplastic potential has an elliptical shape that may grow in size and also rotate in stress space.

Specific details and the mathematical formulation are provided later in this paper. However, it should be emphasized here that many natural soils initially undergo one-dimensional consolidation under the weight of any overburden. If such consolidation were to occur over the reference time, *T*, then the normal consolidation curve labelled '*Ko*-ncl' in Figure 1 would be followed. In these circumstance the stress state at time *T* would be represented by point A in Figure 1a and the state of the soil by point A´ in Figure 1b. For times larger than the reference time *T*, creep deformations would occur under the constant weight of the overburden (constant effective stress), typically from point A´ to A in Figure 1b. If the initial consolidation had occurred under isotropic stress and strain conditions then the path labelled 'ISO-ncl' in Figure 1b would have been followed.

## 2.2 *Mathematical formulation*

For simplicity, the model is presented in terms of the triaxial stress invariants $p'$ and $q$, defined as:

$$p' = \frac{1}{3}\left(\sigma'_1 + 2\sigma'_3\right) \tag{1}$$

$$q = \left(\sigma'_1 - \sigma'_3\right) \tag{2}$$

where $\sigma'_1$ and $\sigma'_3$ are the major and minor principal effective stress components, respectively. In this description, the volumetric and deviatoric strain components are defined respectively as follows:

$$\varepsilon_v = \varepsilon_1 + 2\varepsilon_3 \tag{3}$$

$$\varepsilon_d = \frac{2}{3}\left(\varepsilon_1 - \varepsilon_3\right) \tag{4}$$

where $\varepsilon_1$ and $\varepsilon_3$ are the principal strain components.

The total strain rate is separated into elastic (recoverable) and viscoplastic (time-dependent irrecoverable) components, so that the volumentric and deviatoric components of the total strain rate may be written as follows:

$$\dot{\varepsilon}_v = \dot{\varepsilon}_v^e + \dot{\varepsilon}_v^{vp} \tag{5}$$

$$\dot{\varepsilon}_d = \dot{\varepsilon}_d^e + \dot{\varepsilon}_d^{vp} \tag{6}$$

where the subscripts '*v*' and '*d*' refer to volumetric and deviatoric components, respectively, and the superscripts '*e*' and '*vp*' refer to elastic and viscoplastic components of strain and strain rate, respectively.

## 2.3 *Elastic strain rates*

It is assumed that the elastic strain increments d$\varepsilon_1$ and d$\varepsilon_3$ instantaneously accompany the changes in effective stress d$\sigma'_1$ and d$\sigma'_3$. The relationships between the time rates of change of the stresses and elastic strains may thus be written as:

$$\dot{\varepsilon}_v^e = \frac{\kappa \dot{p}'}{Vp'} \tag{7}$$

$$\dot{\varepsilon}_d^e = \frac{\dot{q}}{3G} \tag{8}$$

where $G$ is the elastic shear modulus of the soil, $\kappa$ is the slope of the unloading-reloading line in $V$-ln$p'$ space and $V$ is the specific volume of the soil.

## 2.4 *Viscoplastic strain rates*

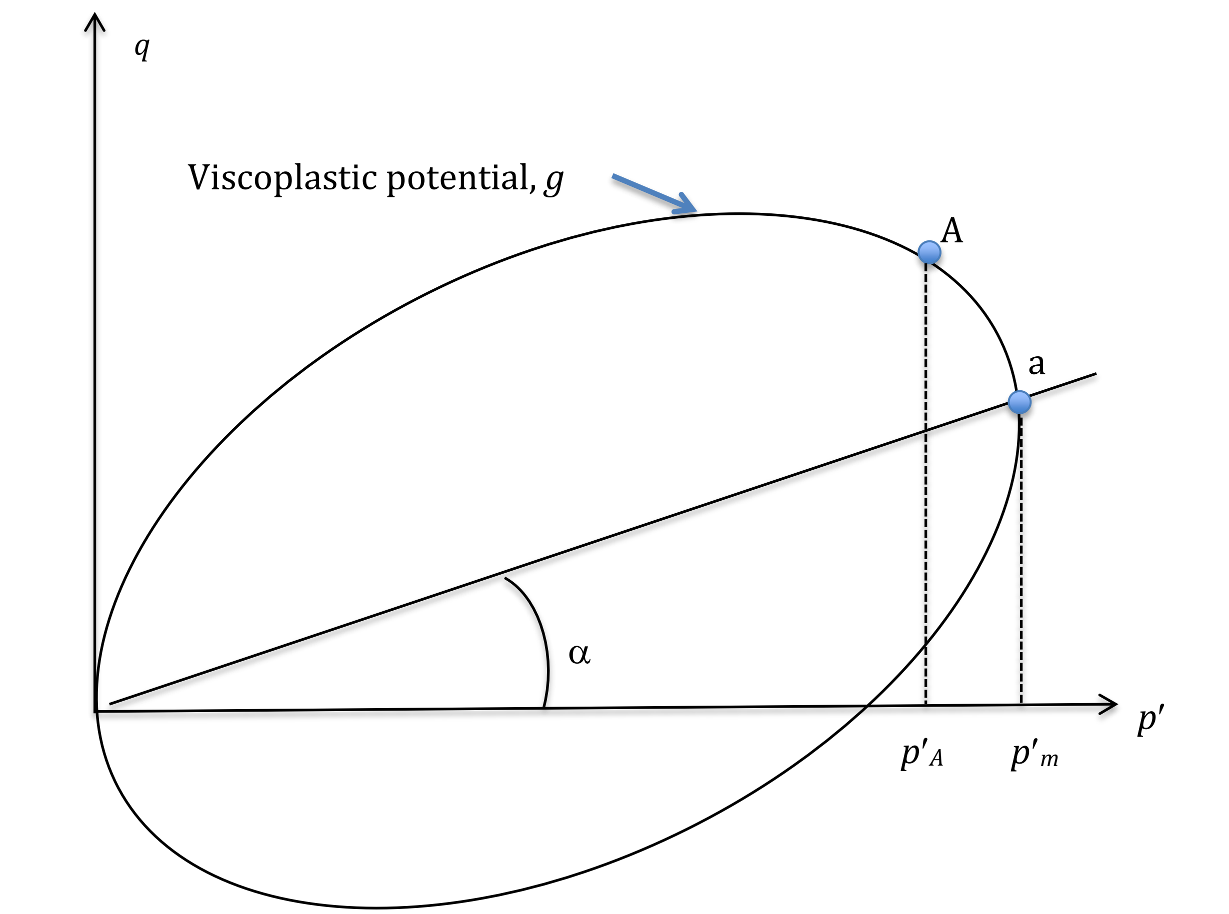
Following Perzyna (1963), Kelln et al. (2008), Sivasithamparam et al. (2015) and others, the viscoplastic strain rates are written in terms of the gradients of a plastic potential surface in stress space, $g$, i.e.,

$$\dot{\varepsilon}_v^{vp} = S\frac{\partial g}{\partial p'} \tag{9}$$

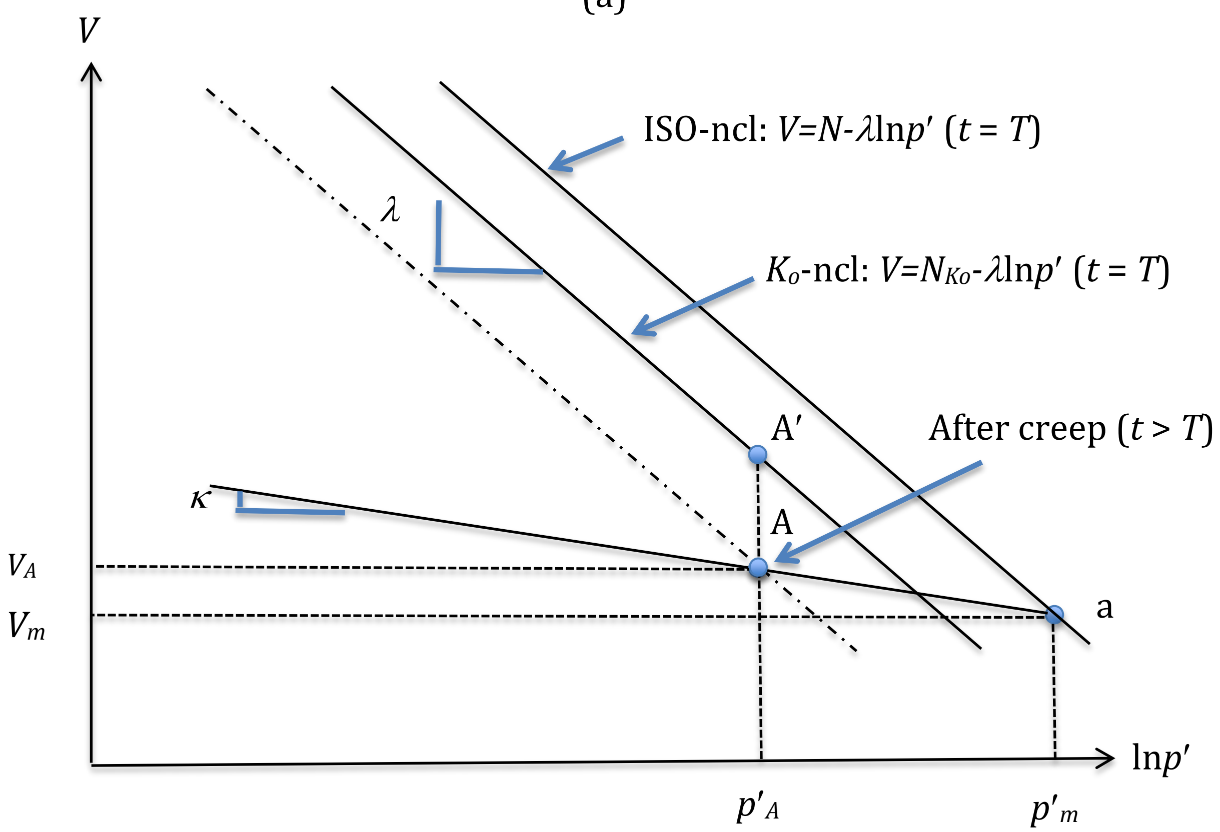
$$\dot{\varepsilon}_d^{vp} = S\frac{\partial g}{\partial q} \tag{10}$$

where $S$ is a scalar multiplier and $g$ plots as an ellipse in stress space (Figure 1a), which can be defined as:

$$g = \left(q - \alpha p'\right)^2 - \left(\mathrm{M}^2 - \alpha^2\right)\left(p'_m p' - p'^2\right) = 0 \tag{11}$$




(a)




(b)

*Figure 1 Schematic representation of anisotropic creep model*

As indicated by Kelln et al. (2008), a suitable expression for the scalar $S$ is given by:

$$S = \frac{\psi}{V_m T}\exp\left(\frac{V_m - N}{\psi}\right)\left(p'_m\right)^{\left(\lambda/\psi\right)}\frac{1}{\left|\frac{\partial g}{\partial p'}\right|} \quad (12)$$

where $\lambda$ is the slope of the normal compression line in $V$- ln$p'$ is space, $\psi$ is the slope of the secondary compression line in $V$-ln$t$ space, and $N$ is the specific volume at unit pressure on the isotropic normal compression line and $T$ is a reference time (usually assumed as 1 day). The specific volume $V_m$ corresponding to $p'_m$ on the particular unloading-reloading line (point a in Figure 1) is given by:

$$V_m = V_A - \kappa \ln\left(\frac{p'_m}{p'_A}\right) \quad (13)$$

where $V_A$ is the actual specific volume of the soil corresponding to the effective stress state $p'_A$-$q_A$ (point A in Figure 1).
However, unlike Kelln et al. (2008), an anisotropic plastic potential is adopted in stress space in this study, as illustrated in Figure 1, for which it can be shown that the gradients of the plastic potential are equivalent to:

$$\frac{\partial g}{\partial p'} = -2\alpha q + 2\alpha^2 p' - \left(M^2 - \alpha^2\right)\left(p'_m - 2p'\right) \quad (14)$$

$$\frac{\partial g}{\partial q} = 2q - 2\alpha p' \quad (15)$$

### 2.5 *Evolution of anisotropy*

The inherent and evolving anisotropy in the response of this ideal soil to changes in stress also has to be defined. In other words, the evolution of the anisotropy parameter $\alpha$ (Figure 1a) must be specified. Following Yang and Carter (2015) and Yang et al. (2015) it is assumed that the parameter $\alpha$, defining the angle of inclination of the elliptical viscoplastic potential to the hydrostatic stress axis, will change from its initial value, $\alpha_0$, with plastic straining. This variation is defined as:

$$\dot{\alpha} = \omega(\eta)\left(\alpha_E(\eta) - \alpha\right)\dot{\varepsilon}_r^p \quad (16)$$

where $\omega$ is a function of the stress ratio $\eta = q/p'$, defined as:

$$\omega = \exp\left(k_g\left(\frac{\left|\alpha - \alpha_E\right|}{M}\right)^{\left|\frac{\eta - \alpha_E}{M}\right|}\right) \quad (17)$$

where $k_g$ is a model parameter and $\dot{\varepsilon}_r^p$ is the rate of plastic strain defined as:

$$\dot{\varepsilon}_r^p = \sqrt{\frac{\left(\dot{\varepsilon}_v^{vp}\right)^2 + \left(\dot{\varepsilon}_d^{vp}\right)^2}{2}} \quad (18)$$

This formulation assumes that volumetric and deviatoric plastic straining contribute equally to the rate of change of the parameter $\alpha$. The parameter $\alpha_E$ is the "equilibrium" value of $\alpha$, which would be achieved after traversing a given stress path at constant stress ratio for sufficient distance. Its value is a function of the stress ratio $\eta$ and can be calculated from the following equation:

$$\frac{\alpha_E}{M} = \frac{\eta}{M} + \frac{M}{2\zeta}\left(1 - \left(\frac{\eta}{M}\right)^2\right)\ln\left(1 - \frac{\eta}{M}\right) \quad for\ \frac{\eta}{M} < 1 \quad (19)$$

where $\zeta$ is also a model parameter.

### 2.6 *Strain increments*

The elastic and viscoplastic strain increments may now be recovered by integrating equations (7) - (10) over time, after noting the particular expressions in equations (12) – (19).

## 3 NUMERICAL PREDICTIONS

### 3.1 *Cases Considered*

Predictions are provided here for the undrained triaxial compression of Saint

Herblain clay loaded at different rates of axial strain during the same compression test. The experimental data for this material have been extracted from Yin and Hicher (2008), who reported the original source of the data as Rangeard (2002). Some of the same experimental data was also cited in a subsequent paper by Yin et al. (2010).

A single triaxial test is simulated in which the axial strain rate was initially 1%/hour and then reduced to 0.1%/hour, increased to 10%/hour and finally reduced again to 0.1%/hour. The experimental data for this case are plotted in Figures 2 to 4.

### *3.2 Model Parameters*

The values selected for the model parameters in the simulation are listed in Tables 1 and 2. Justification for their choice is provided below.

*Table 1 Assumed material properties.*

| Parameter | Definition | Value |
|---|---|---|
| M | Friction constant | 1.25 |
| $\lambda$ | Slope of ncl in $V$-ln$p'$ space | 0.5 |
| $\kappa$ | Slope of url in $V$-ln$p'$ space | 0.06 |
| $\nu$ | Poisson's ratio | 0.3 |
| $N$ | Location of Iso-ncl in $V$-ln$p'$ space | 5 |
| $\psi$ | Slope of secondary consolidation line in $V$- ln$t$ space | 0.03 |
| $\zeta$ | Anisotropy parameter | 1 |
| $k_g$ | Anisotropy rotation parameter | 2 |
| $T$ | Reference time (days) | 1 |

*Table 2 Assumed initial conditions.*

| Parameter | Definition | Value |
|---|---|---|
| α□ | Initial value of $\alpha$ | 0.2 |

The values of the friction parameter for triaxial compression, M □ □□□□, and Poisson's ratio, $\nu$ = 0.3, are as suggested by Yin and Hicher (2008), although it is curious that in their later paper, Yin et al. (2010), they adopted slightly different values for these parameters (1.2 and 0.2 respectively). Values of 0.5 and 0.06 have been adopted for the parameters $\lambda$ and $\kappa$ respectively. These values have been interpreted independently, from the results of oedometer test reported by Yin et al. (2010), who previously reported a value of 0.48 for $\lambda$ and 0.022 and 0.038 for $\kappa$.

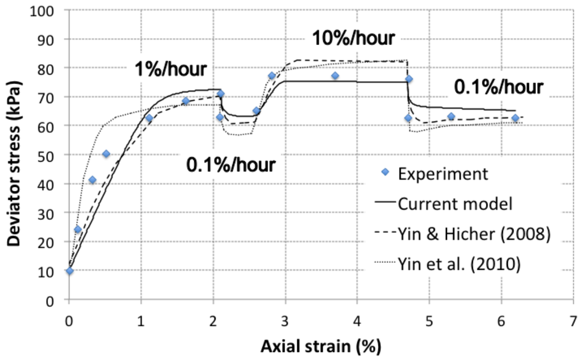



*Figure 2 Undrained triaxial test on Saint Herblain clay – Deviator stress versus axial strain.*

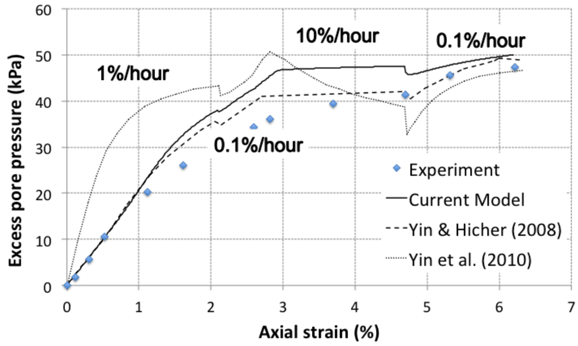



*Figure 3 Undrained triaxial test on Saint Herblain clay – Excess pore pressure versus axial strain.*

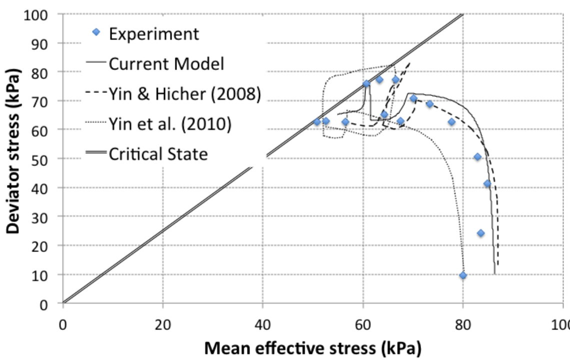



*Figure 4 Undrained triaxial test on Saint Herblain clay – Stress paths.*

The predictions of the current model are somewhat sensitive to the value of $\kappa$ □□□□□□□□□□□□□□□□□□□□□□□□□□□□□□

□□□□□□□□□□□□□□□□□□o to some degree the choice here has been influenced by how well the model was able to simulate the experimental results. However, based on the available experimental data, it is noted that the value of $\kappa$ is open to some interpretation and a value of 0.06 would seem to fit into the range of most likely values, although probably at the high end.

The value of the parameter $N$, determining the position of the isotropic consolidation line in $V$- ln$p'$ space was determined from the available consolidation test data (as published in Yin et al. 2010).

Figure 12 of Yin et al. (2010) provides a plot of voids ratio, $e$ (= $V$ - 1), versus time plotted on a base 10 logarithmic scale. It also indicates interpreted values of 0.0337 and 0.0341 for the so-called secondary compression index, $C_{\alpha e}$, defined by the following relationship

$$C_{\alpha e} = \frac{\Delta e}{\Delta \log_{10}(t)} \tag{20}$$

The parameter $\psi$ is related to $C_{\alpha e}$ as follows:

$$\psi = \frac{C_{\alpha e}}{\ln(10)} = \frac{C_{\alpha e}}{2.303} \tag{21}$$

The value of 0.03 adopted here for the parameter $\psi$ is therefore approximately twice the value that would be interpreted from Figure 12 of Yin et al. (2010) using equation (21). However, it is considered to be of about the correct order and possibly within the range of values that might be interpreted from the available data. Again, it is noted that the choice here has been influenced by how well the model was able to simulate the experimental results for Saint Herblain clay in undrained triaxial compression.

As noted by Yang et al. (2015), virgin consolidation tests at constant stress ratio constitute the cornerstone for describing the effect of plastic anisotropy on the soil fabric. Thus the parameters $\zeta$ and $k_g$ are best determined by fitting the model to the behaviour of the soil as measured in such test. However, no such data was readily available for Saint Herblain clay and so the values of these parameters were determined by assessing how well the simulations fitted the data in the undrained triaxial tests.

The value of the initial inclination of the plastic potential surface in stress space, $\alpha_i$ = 0.2, was similarly assessed, due to a lack of appropriate test data.

In summary, where there existed appropriate laboratory test data, these data were used to independently determine the values of the model parameters. Where such data was unavailable, resort was made to curve-fitting the undrained triaxial compression tests. This situation is not entirely satisfactory from the point of view of independently assessing the merits or otherwise of any constitiutive model. Nevertheless, this situation is not uncommon in geotechnics.

## 4 PREDICTIONS OF OTHER MODELS

Simulations of the undrained triaxial compression response of the Saint Herblain clay, using other published models, are also provided here for comparison purposes. These correspond to the different elasto-viscoplastic stress-strain models proposed by Yin and Hicher (2008) and Yin et al. (2010). The numerical predictions have been extracted from figures in the source references.

### *4.1 Model of Yin and Hicher*

The model proposed by Yin and Hicher (2008) is an elasto-viscoplastic model based on Perzyna's overstress theory and on the elasto-plastic Modified Cam Clay model. This model assumes isotropic behaviour and it would appear from the description provided in the paper by Yin and Hicher (2008) that a constant value of 5,000 kPa may have been assumed for Young's modulus of the clay. If this is correct, then such behaviour would be inconsistent with the usual implementation of Modified Cam Clay, for which it is usually assumed that either the elastic shear modulus or Poisson's ratio is constant but the elastic bulk modulus is pressure (and voids ratio) dependent.

The model of Yin and Hicher differs from the model presented here in at least other two important ways. First, the model proposed in this paper involves anisotropy in the viscoplastic response, whereas the Yin and Hicher model assumes isotropy. Second, the viscoplastic potential function is different in the two models. The model proposed here adopts the potential defined by equation (11) and the scalar multiplier given by equation (12). However, Yin and Hicher have assumed the Modified Cam Clay ellipse as the potential function and a scalar multiplier, *S*, given by:

$$S = \mu\left[\exp\left(N\left(\frac{p_c^d}{p_c^s} - 1\right)\right) - 1\right] \qquad (22)$$

where $\mu$ and $N$ are viscosity parameters assigned values of $\mu = 10^{-9}$/s/kPa and $N = 10$. The parameters $p_c^s$ and $p_c^d$ define the sizes of the so-called static and dynamic loading surfaces.

### *4.2 Model of Yin et al.*

In a later paper, Yin et al. (2010) appear to have extended the model of Yin and Hicher (2008) to include an anisotropic viscoplastic response. This generalisation involves an elliptical viscoplastic potential that can rotate and expand in stress space, much the same as the model presented in this paper but with a differnt law governing that evolution. The latter paper has also adopted a different form for the viscoplastic scalar multiplier, *S*, viz.,

$$S = \mu\left(\frac{p_c^d}{p_c^s}\right)^N \qquad (23)$$

For the predictions made using the model of Yin et al. (2010) the following values were adopted for the new viscosity parameters: $\mu = 7.4\text{x}10^{-8}$/s and $N = 12.9$.
The values of the other parameters assumed in each of these two models are presented and discussed in the source papers and so they are not repeated here.

### *4.3 Commentary*

It is worth noting that the expressions for the scalar multipliers, *S*, adopted in the models of Yin and Hicher (2008) and Yin et al. (2010) (Equations 22 and 23) avoid one of the major difficulties of the model proposed by Kelln et al. (2008) and hence also adopted in the model proposed here (Equation 12). This difficulty arises from the inherent assumption in the model of Kelln et al. that the volumetric component of the viscoplastic strain rate (and therefore also the viscoplastic volumetric strain increment) is constant around the plastic potential surface. As such, the critical state cannot be properly simulated, since no volumetric strain increment should be predicted at critical state. This shortcoming of the model may produce erroneous or inaccurate predictions for stress states at or close to critical state.

## 5 DISCUSSION

The simulations of all three models considered in this paper are presented in Figures 2 - 4 where they are compared with each other and the experimental data obtained for the Saint Herblain clay sample by Rangeard (2002).
From Figure 2 it can be deduced that all three models provide a reasonable simulation of the stress-strain response of the rate-sensitive clay soil, provided suitable parameter values are adopted in each case. In all cases it is clear that the elasto-viscoplastic model framework is able to capture the rate-sensitivity of the response of this soil during undrained triaxial compression. It is arguable which model provides the most accurate simulation.
Figure 3 reveals that all predictions of excess pore pressure generated by the undrained shearing are perhaps not quite as accurate as the corresponding predictions of the deviator stress.
Figure 4 presents the predicted undrained stress paths. Reasonable, but by no means close agreement, is shown here for all model simultions. It is also notable that the initial curvature of the undrianed stress path displayed in the experiments cannot be explained or predicted by any of these models. As noted by Yin et al. (2010), this is likely to be due to anisotropy in the elastic response of the clay, a feature not captured in any of the models considered here.

## 6 CONCLUSIONS

A new constitutive model has been presented that can describe the rate-dependent response of soft clay soils. This model builds on the work of previous researchers essentially combining certain aspects of models published previously by Kelln et al. (2008) and Yang et al. (2015).

It has been shown that the model is able to simulate reasonably the rate-dependent response of Saint Herblain clay in undrained triaxial compression. The noticeable changes in the response of the clay as the strain rate was varied during undrained shearing was captured well.

However, it was also noted that other constitutive models can capture this behaviour, arguably, equally as well. Two additional models were considered in this context and all three models were developed within the framework of elasto-viscoplasticty and overstress theory.

Deciding which particular model should be used in engineering predictions of the rate-dependent response of a given soft clay soil is not a straightforward matter, as a number of models possess similar characteristics and capabilities. The choice may ultimately depend on familiarity with the model and the availability of suitable laboratory or field test data from which the values of the model's parameters may be reliably determined.

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