The influence of the shaft friction and pile shape on the pile tip bearing capacity

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ABSTRACT
In 1920 Prandtl published an analytical solution for the bearing capacity of a maximum strip load on an infinite half-space, based on a sliding soil part, with three sliding zones, which is nowadays called the Prandtl-wedge. This solution was extended by Reissner with a surrounding surcharge. Terzaghi wrote the formula with bearing capacity factors and Meyerhof extended this formula with both inclination factors and shape factors.
Because of these developments for shallow foundations, many geotechnical researchers thought that failure of a pile tip in a deep sand layer will also show a Prandtl-wedge type of failure and that the stresses on a pile tip are also constant and do not depend on the shape and size of the pile tip. This would imply that a Cone Penetration Test (CPT) gives the same stress as a real pile tip, and can be used without a reduction for calculating the bearing capacity of a real pile.
Field tests show however that a bearing capacity of a pile tip, based on unreduced CPT data, is too high. Therefore this problem has been modelled and studied, with Finite Element Modelling.
Several remarkable results were found. There is no Prandtl-wedge type of failure at the pile tip, but a general zone of plasticity. Also the stresses below the pile tip are not constant, but higher near the centre of the pile. The difference in bearing capacity between CPT and real pile does not depend on the shape and size of the pile tip, but more on the difference of the definitions of failure between CPT and pile.
Additional calculations show that the pile shaft friction does not influence the stresses at the pile tip, but the normal stresses on the pile tip do influence the shear stresses along the shaft.

Keywords: Bearing capacity, Pile Foundations, Cone Penetration Test.

1 INTRODUCTION

1.1 Prandtl-Reissner

In 1920, Ludwig Prandtl published an analytical solution for the bearing capacity of a soil under a limit pressure, p, causing kinematic failure of the weightless infinite half-space underneath. The strength of the half-space is given by the angle of internal friction, ϕ , and the cohesion, c. The solution was extended by Reissner in 1924 with a surrounding surcharge, q. Prandtl subdivided the sliding soil part into three zones (see Figure 1):

Zone 1: A triangular zone below the strip load. Here the largest principal stress is in the vertical direction.

Zone 2: A wedge with the shape of a logarithmic spiral, in which the principal stresses rotate through $90°$ from Zone 1 to Zone 3. The pitch of the sliding surface equals the angle of internal friction; $\xi = \phi$, creating a smooth transition between Zone 1 and Zone 3.

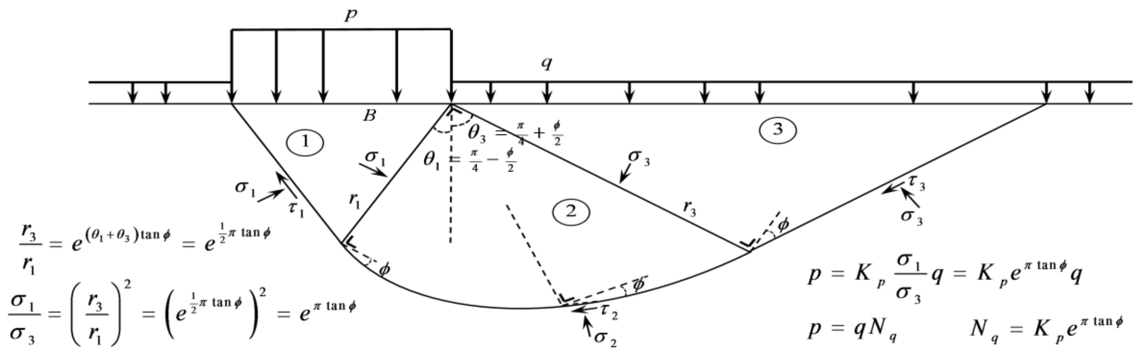

Figure 1 The wedge of Prandtl.

Zone 3: A triangular zone adjacent to the strip load. Here the largest principal stress is in the horizontal direction.

1.2 Meyerhof

Keverling Buisman (1940) and Terzaghi (1943) extended the Prandtl-Reissner formula for the soil weight, γ. And in 1953 Meyerhof was the first to propose equations for inclined loads. He was also the first in 1963 to write the formula for the (vertical) bearing capacity p_v with bearing capacity factors (N), inclination factors (i) and shape factors (s), for the three independent bearing components; cohesion (c), surcharge (q) and soil-weight (γ), in a way it is still used nowadays:

$$p_v = i_c s_c c N_c + i_q s_q q N_q + i_\gamma s_\gamma \tfrac{1}{2}\gamma B N_\gamma . \quad (1)$$

1.3 Pile tip

Because of this analytical solution for shallow foundations, many researchers (for example Meyerhof, 1951) thought that a pile tip in a deep sand layer will also show a Prandtl-wedge type of failure mechanism and equation 1 can be used.

In this article only vertically loaded deep pile tips are considered (so the influence of the soil weight can be neglected), in cohesionless soils, which means that this equation can be reduced to:

$$p = s_q q N_q , \quad (2)$$

in which the surcharge bearing capacity factor is given as (see figure 1):

$$N_q = K_p \cdot \left(\frac{r_3}{r_1}\right)^2 = K_p \cdot e^{\pi\tan\phi} \quad (3)$$

with: $K_p = \dfrac{1+\sin\phi}{1-\sin\phi}$.

This means that the stresses on the pile tip will also be constant and depend only on the friction angle of the soil, the vertical effective stress near the pile tip, and its shape factor and not on the shape and size of the pile tip. This would imply that a Cone Penetration Test will produce the same average stress as a real pile and can in principle be used without a reduction or a scale factor for the calculation of the bearing capacity of a pile, just as Boonstra (1940) showed with his field tests and just as is assumed in many pile bearing capacity predicting models, such as the model of Van Mierlo & Koppejan (1952).

1.4 Validation

The solution of the surcharge bearing capacity factor N_q (equation 3) has been checked by Van Baars (2015) with finite element calculations, and found to be correct. But this does not proof that the Meyerhof equation can also be used for pile tips, because the analytical solution is based on a Prandtl-wedge failure mechanism, while for circular shallow foundations and pile foundations (see also paragraph “4 Failure mechanism”) another failure mechanism is found. Besides, the finite element calculations also show that the solutions of the surcharge shape factor of both Meyerhof (1963) and also De Beer / Brinch Hansen (1970) are not accurate and should be more like (see Tapper et al, 2015):

$$s_q = 1 - 0.55 \cdot \sqrt{B/L}\,, \qquad (4)$$

in which B and L are the width and depth of the pile tip, so for square pile tips: $B / L = 1$.

Another big problem is that many researchers (Jardine et al, 2005, Lehane et al, 2005, Clausen et al, 2005) and recent field tests (Van Tol et al., 1994, 2010, 2012) show that a calculated bearing capacity based on unreduced Cone Penetration Test data, can be more than 30% higher than measured.

2 FINITE ELEMENT MODELLING

In order to find the reason for all this, a pile has been modelled and studied. With the software code Plaxis, displacement controlled, non-updated mesh, 2D axial-symmetric, finite element calculations have been made. In all cases a 10 m deep pile in dry sand has been modelled with a Mohr-Coulomb model, using the soil parameters listed in table 1.

Table 1 Soil parameters.

Parameter	Value
Friction angle	$\phi = 35°$
Dilatancy angle	$\psi = 0°$
Cohesion	$c = 0$
Young’s modulus	$E = 50{,}000$ kPa
Poisson’s ratio	$\nu = 0.3$
Unit weight	$\gamma = 20$ kN/m^3

Higher-order 15-node elements have been used. The element sizes can be seen in Figure 4.
The idea is to keep the calculation as simple as possible so that differences between two calculations are caused by the change of a pile size, and not due to second order effects of for example a complex soil model. Therefore the pile is a wished-in-place pile; no installation effects are added. For the same reason, no large strain corrections have been made, even though large deformations were applied; First because Plaxis cannot model correctly the flow of a plastic soil, around a pile tip at large displacements. Second because not the exact stresses are sought, but the failure mechanisms and the relative differences between two or three options.

Both the pile tip and the pile shaft have been modelled by a displacement controlled line or soil boundary, without any interface. This represents an infinite stiff and rough pile. For each calculation step an additional pile displacement of 1 mm has been chosen.
The advantage is of this method is that this calculation is very simple and very stable, and the push in force is simply the same as the total reaction force which is standard registered.
The disadvantage is that it is unknown how much of this force comes from the tip resistance and how much from the shaft.

3 SHAPE AND SIZE OF PILE TIP

The first step was to investigate the influence of the shape of the pile tip. First four almost identical calculations have been made: two calculations with normal (flat) pile tips and two with sharp pile tips (pile tip angle $\alpha = 45°$). Two of them were calculations with only the pile tip pushed down and two with both the tip and shaft pushed down (vertical displacement controlled boundary; no horizontal displacements).
Figure 2 shows that the difference in force between a sharp and a normal (flat) pile tip can be neglected. Also three different pile diameters have been tested.
Since the circumference of the shaft is scaled in a different way as the area of the pile tip, only the pushing down of the pile tip has been modelled here (so with free displacements for the shaft).

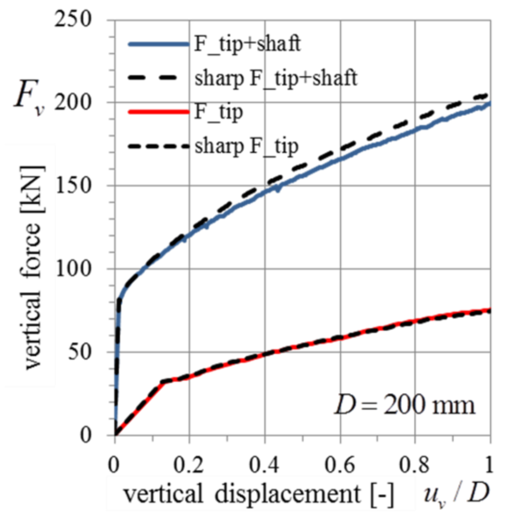

Figure 3 Influence of the shape of the pile tip

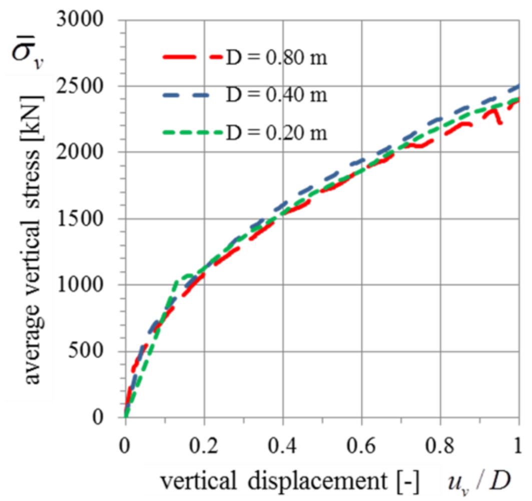

Figure 2 Influence of the pile tip diameter.

Figure 3 shows the average vertical stress below the pile tip $\bar{\sigma}_v$ versus the vertical displacement u_v divided by the pile tip diameter *D*, proofing that the curves are the same for all three pile diameters, just as the Prandtl solution would predict.

3.1 Continuously growing stresses

The phenomenon of the continuously growing stresses with displacement is very important, since the cone resistance is defined for a (failing) pile tip at an infinite displacement, while the bearing capacity of a real pile is defined for a pile tip at only 10% displacement of (the size of) the pile tip. This difference in definition of failure explains at least a substantial part of the 30% difference of the stresses at the pile tip between the cone (at infinite displacement) and a pile (at only 10% displacement).

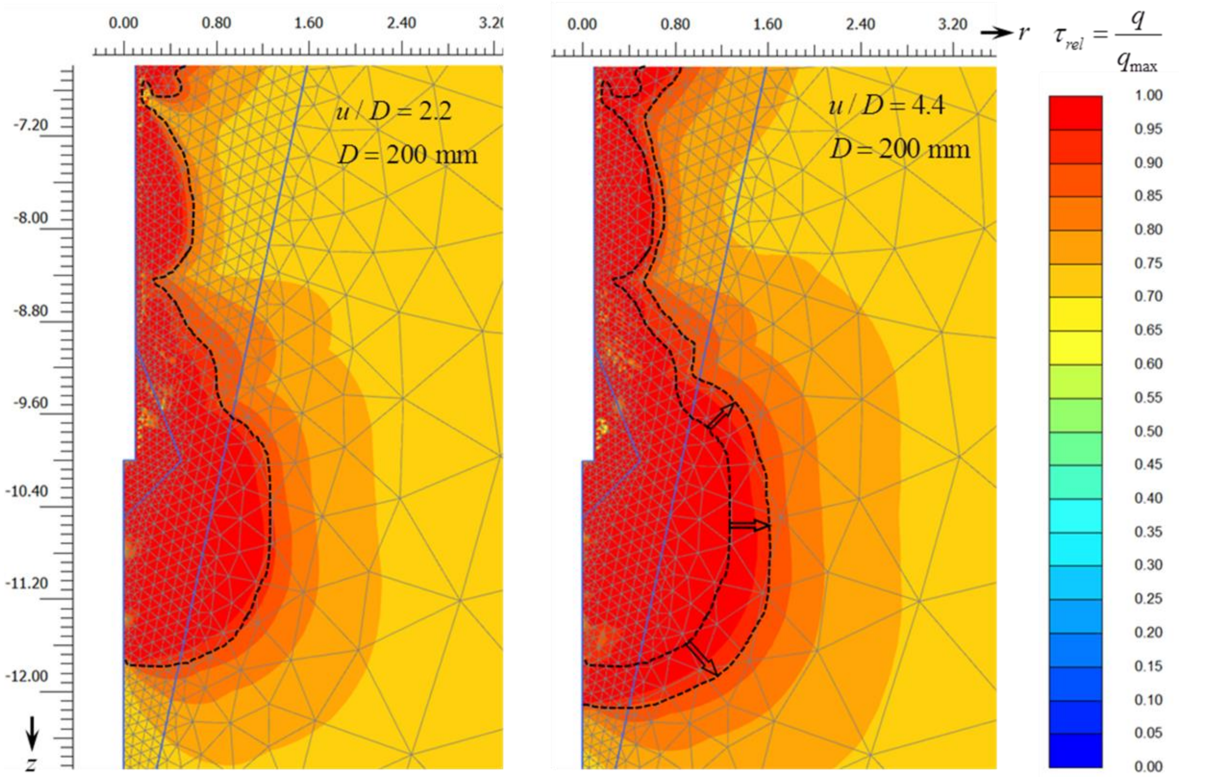

Figure 4 Failing mechanism: growing total plastic zone.

4 FAILURE MECHANISM

Figure 4, on the previous page, shows the relative shear stress τ_{rel} after large pile displacements u_v; 2.2 times (on the left) and 4.4 times (on the right) the pile diameter D. This relative shear stress is defined as the radius of the circle of Mohr divided by the radius of a circle touching the Coulomb line, which means plasticity or failure of the soil, or a relative shear stress of "1".

The failure zone in this figure is not like a Prandtl-wedge. In fact there is an ever growing plastic zone below the pile tip and all along the pile shaft. The fact that the zone is growing, means that the vertical stress (the bearing capacity) is also growing with the displacement. This ever growing bearing capacity was already noticed from figure 2, but follows also from figure 5, which shows the push-in bearing capacity of the shaft and the tip individually (2nd and 3rd line from below), but also together (2nd line from above). Two other lines show the pull-out force of the shaft alone (lowest line) and the summation of the tip push in and the shaft pulled out (3rd line from above).

The highest (dashed) line is found by simply summing up the push-in force of both the pile tip and the pile shaft.

The 3rd line from above is found by summing up the push-in force of the pile tip and the pull-out force of the pile shaft. One could think that for a calculation with both the pile tip and the shaft moving down at the same time, the upper dashed line will be found, but instead the bearing capacity (2nd and straight line) has been found to be clearly lower. This proofs that the pile shaft and tip influence each other a lot, causing a strong non-linearity or reduction in the bearing capacity.

The question arises if this reduction of the bearing capacity exists because the shaft influences the pile tip or because the tip influences the shaft, or maybe even both. This question will be solved in the next paragraph.

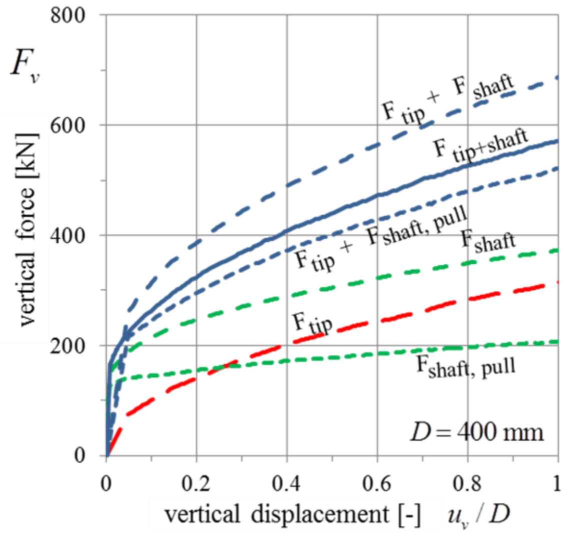

Figure 5 Bearing capacity of the shaft and the tip.

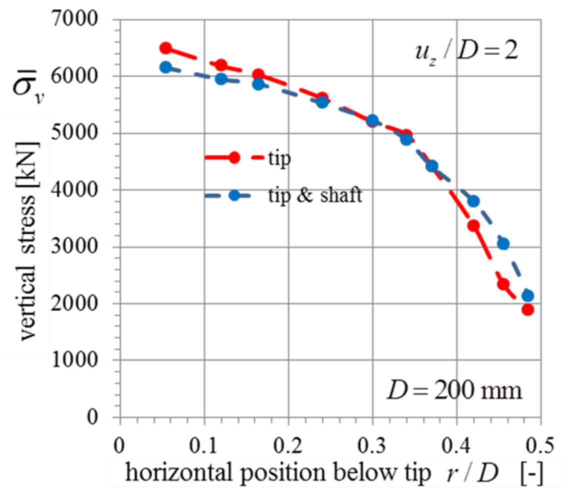

Figure 6 The normal stress below the pile tip.

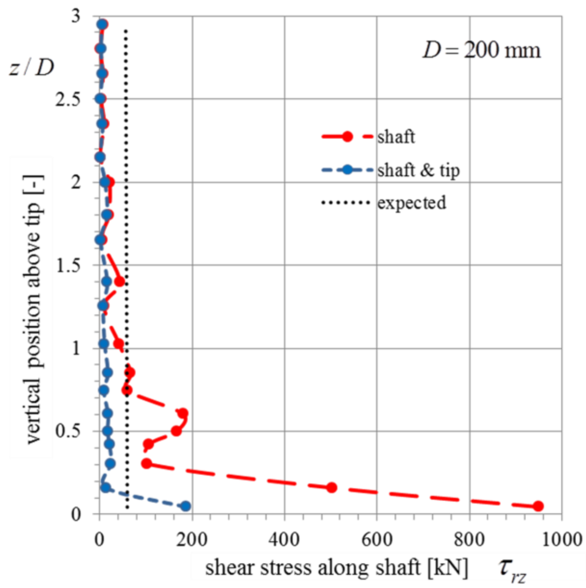

Figure 7 The shear stress along the shaft.

5 STRESSES ALONG THE TIP AND SHAFT

The previous calculations were displacement controlled, which means that only a single total force results from the calculations, and not two independent forces: the force at the tip and the force along the shaft.
Therefore the normal stresses along the (flat) tip have been plotted (Figure 6) and also the shear stresses along the shaft (Figure 7), for both a calculation in which only the tip has been pushed down and also a calculation in which both tip and shaft have been pushed down for a large displacement ($u_z / D = 2$).
These figures show that the normal stresses below the tip are (almost) not influenced by the shaft, but that the shear stresses along the shaft are influenced by the tip.
Figure 6 also shows that the normal stresses below the pile tip are not constant, but higher near the centre of the pile, unlike the Meyerhof equation.
The dotted line in Figure 7 named “expected” is the shear stress related to the vertical effective stress σ'_v and the horizontal earth pressure K_0, according to:

$$\tau_{rz} = \tan\phi \cdot \sigma_v \cdot K_0 , \qquad (5)$$

with: $\sigma'_v = \gamma' z$ and: $K_0 = 1 - \sin\phi$.

Figure 7 shows that most of the pile shaft has less shear friction than expected, except near the tip, but also that the pile tip has a big influence on the shaft: the pile tip is pushing down the surrounding soil and reduces in this way the shear stresses along the shaft.

6 CONCLUSIONS

According to field tests, large differences are found between the normal stresses on the cone in a CPT test, and the normal stresses on the tip of a foundation pile, while a good scientific explanation cannot be given. Also the influence of the pile size (scale effect), the influence of the installation (driving or pushing), the influence of the pile tip on the pile shaft and visa-versa and the influence of the horizontal soil stresses are not completely understood. In fact even the type of failure mechanism around the pile tip is still a point of discussion.
A conclusion following from the numerical calculations is that, the shear stresses along the pile shaft do not influence the stresses at the pile tip, but visa-verso the normal stresses at the pile tip do influence the shear stresses along the shaft.
Another conclusion from the numerical calculations, and as expected from the Prandtl theory, is that the shape and size of the pile tip do not matter. But according to this research, there is no Prandtl-wedge type of failure at the pile tip, but a zone of plasticity. And the stresses below the pile tip are not as given by the Prantl theory constant, but higher near the centre of the pile.
The outcome that the shape of the pile has no influence is already what is used in practise; the stresses measured with a sharp cone are assumed to be the same for a flat tip of a foundation pile.
The outcome that the size does not matter might look in contradiction with the field tests, but before concluding this, first future tests should compensate for the difference in failing definition (the 10% displacement rule for a real pile versus the infinite displacement for a Cone Penetration Test).

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