A geostatistical analysis of variations of permeability within a compacted dam core

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ABSTRACT

The spatial variability of the geotechnical properties of compacted till core embankment dams was studied using geostatistical analysis in the past and showed promising results. The main goal of this study is to improve the geostatistical method used to study such parameters. A quasi 3D approach using a 2D interpolation grid representing the entire core and focusing on the assessment of the spatial distribution of permeability within the core of a dam located in north-eastern Québec is presented in this paper. Geostatistics were used as a useful means to enhance the value of construction control data, from which permeabilities were calculated using empirical relationships based on the clay fraction and the fabric of the compacted tills. The study shows that the core's structure is not only stratified according to elevation, but can also vary significantly within a single lift and that this variability is strongly linked to the fabric of the compacted till. The geostatistical method used in this paper shows great potential and versatility as it can be used to model geotechnical properties of an embankment dam even before its impoundment.

Keywords: Embankment dam, Geostatistics, Kriging, Permeability, Construction control data

1 GUIDELINES

The hydraulic barrier performance of compacted till cores of embankment dams is highly dependent on compaction conditions during construction, which alongside basic geotechnical properties may significantly vary spatially. The recent development of new tools and a better understanding of the variability of soil properties enable the integration of such variability in design and behaviour analysis using geostatistics as a good analysis tool.

As geotechnical modelling and analysis projects often proves very costly, a very promising path is to develop new methods to extract a maximum of information from already existing data such as construction control data.

Past studies have shown that geostatistics can be used as a very interesting tool to assess and study the spatial variability of the geotechnical properties of dam materials (Smith and Konrad, 2011; Smith, 2002; Soulié et al. 1983), such as permeability, using georeferenced construction control data. These studies led to advances in the understanding of various issues related to the stratified nature of dams' structures and broadened perspectives for geostatistical analyses in large embankment design.

This study focuses on the assessment of the permeability, based on the analysis of the construction control data of an embankment dam. Permeabilities were estimated based on the aggregated or homogeneous fabric of the compacted till to reflect the significant vulnerability of dams to changes in the compaction conditions during construction. Geostatistics and kriging were used to study the spatial variability and to detect higher permeability areas that could prove potentially problematic. Kriging is a linear estimation method which, as opposed to other methods like weighted averages and classical regression, takes into account the spatial dependency of the studied parameters. The method was first developed for the estimation of mineral potential in mining engineering (Krige, 1951) and later formalized as a more general statistical method (Matheron, 1963), it is now widely used in various fields as a means to analyze spatially correlated data.

The geostatistical approach presented in this paper is based on the spatial coordinates of the sampling points on a 2D plane representing the entire 3D structure of the dam's core. The whole project will study the impact of performing geostatistical analysis on the estimated values of permeabilities for the various sampling points compared to the parameters that govern permeability. This paper will deal with the analysis of geostatistical results performed on the estimated permeabilities at the various sampling points.

2 TEST SITE

The structure analysed in this project is a compacted till core dam from a hydro-electrical complex in north-eastern Québec. The complete structure of the dam has a maximum height of 171 m, a crest elevation of 410 m, a crest length of 378 m, a crest width of 10 m and a total volume of 6 300 000 m^3. The core itself has a maximum elevation of 169.2 m, a crest elevation of 408.2 m, a crest length of 378 m, a crest width of 4 m and a volume of approximately 905 000 m^3. The structure was built by laying 0.5 m thick lifts following a regular pattern from the right bank to the left bank and back. The important number of construction surveillance data and its regular construction sequence makes this dam the perfect candidate for geostatistical analysis. The layout of the dam's structure is showed on Fig. 1, where the core is highlighted in white color over the grey shaded 3D graph.

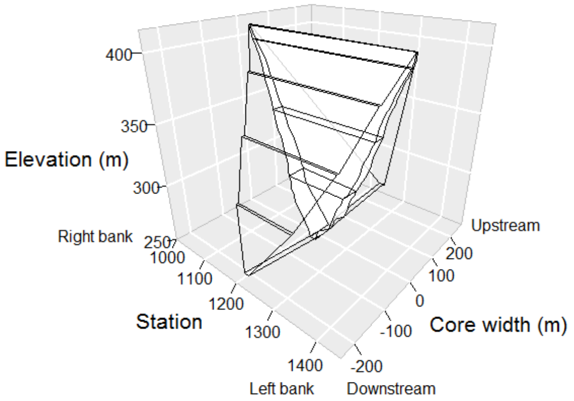

Figure 1 - Geometry and dimensions of the dam

The tills used for the construction of the core come from five borrow pits (DE-9, DE-9A, DE-9B, DE-9Est and DE-12) located around the construction site and presented variable geotechnical properties. Since the focus of this study is to assess the permeability of the core and because this parameter is strongly affected by the distribution of fine content, these properties are presented in Tab. 1 for each borrow pit.

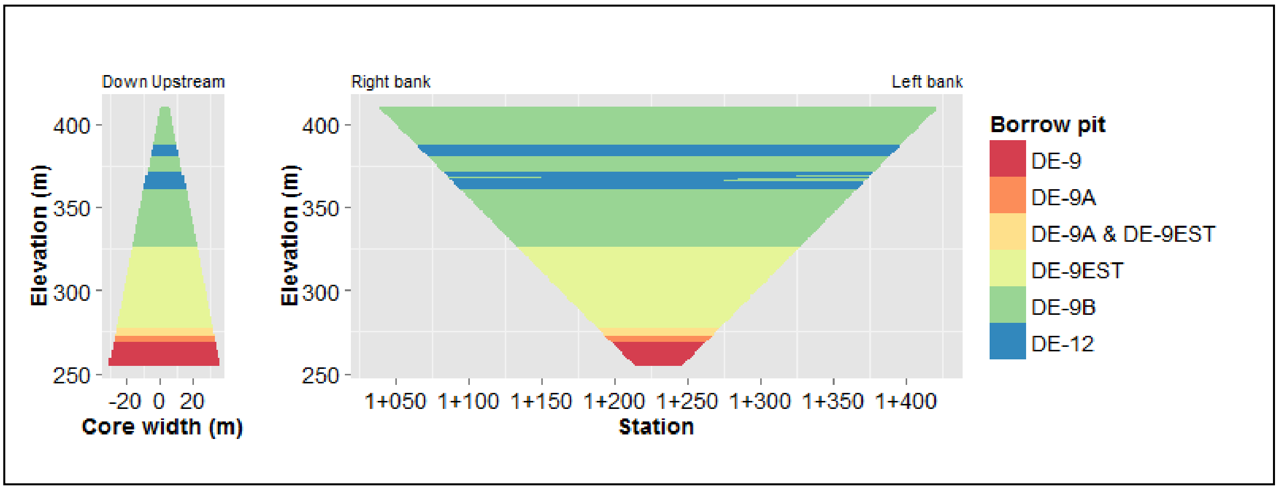

Figure 2 - Core materials distribution

The location of the tills within the core's structure is given in Fig. 2.

Most of the core was built using tills from borrow pit 9, 9A, 9B and 9Est. Those borrow pits are in fact different faces of a same excavation site and show similar grain size distribution. The fines fraction (% passing < 80 µm) varies from 24% to 53% and the clay fraction (% passing < 2 µm) from 1.8% to 8.1%. Tills from borrow pit 12 were used only for a short time and its use was limited to a small part of the core showed in blue around elevations 325 and 360 m in Fig. 2. This borrow pit was exploited because it was closer to the construction site and presented an economic benefit. The use of this till was stoped because of its weak geo-mechanical properties. For example, it had significantly higher fines and clay fractions, 30% to 69% and 2.7% to 11.1% respectively, than the other borrow pits and would lead to weaker strain resistance while resulting in a much lower permeability.

Table 1 – Till fine content distribution

Borrow pit	**Fines fraction, % < 80 µm**			**Clay fraction, % < 2 µm**		
	min	**max**	**av.**	**min**	**max**	**av.**
DE-9	37	48	43	2.5	5.0	3.1
DE-9A	31	53	37	2.5	8.1	4.5
DE-9B	24	48	34	1.9	6.2	3.0
DE-9E	26	46	37	1.8	5.4	2.8
DE-12	30	69	54	2.7	11.1	7.1

3 GEOSTATISTICS

A simplified flowchart of the geostatistical approach used for this study is presented in Fig. 3 which is detailed in sections 3.1 to 3.4.

The geostatistics analysis computing was realized using the statistically oriented *R* programming language alongside the *gstat* package which includes many useful geostatistics related tools.

3.1 Exploratory statistics

The first step of any geostatistical process should concentrate on the analysis of the available data with simple statistics. This step gives important information about the nature of the data set distribution and about the feasibility of a geostatistical analysis on these data. For example, the study of a data set distribution can highlight the need for a data transformation. Extreme values have an important impact on the variogram and parameters such as permeability, which often shows highly skewed distributions, need to be transformed in a way that allow their use as part of a variographic analysis. In these cases, one of the most commonly used transformations is the logarithm ($\log_{10}$) transform, which transforms the data into a form closer to a gaussian distribution. The next steps of the analysis should be realized on the transformed data.

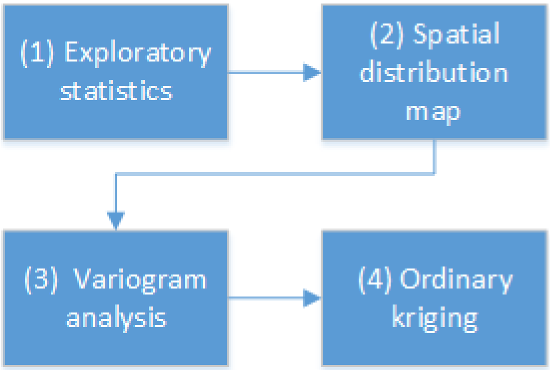

Figure 3 - Geostatistics flowchart

3.2 Spatial distribution map

The goal of this step is to produce a distribution map of the samples to appraise the quality of its spatial distribution and to establish if the available data respects the conditions for kriging. A good spatial distribution should contain enough data points to represent the analysed phenomenon but should also be evenly distributed across the studied area and devoid of data cluster of significant size.

3.3 Variogram analysis

A variogram is a function (Eq. 1) used to assess the spatial correlation of a set of spatial random variables relative to the separation distance between the data points (Cressie, 1993).

$$\gamma(h) = \frac{1}{2n(h)} \sum_{i=1}^{n} [V(x_i) - V(x_i + h)]^2 \quad (1)$$

where $\gamma(h)$ is the spatial variability estimator, V a random variable, h the separation distance between two sampling points x_i and x_i+h and $n(h)$ the number of distinct pairs separated by the lag distance h.

The spatial variability estimation is generally an increasing function which can reach one or more plateau. The first of these plateaus is called the sill and its value is generally very close if not equal to the variance of the studied parameter. The separation distance h at which the sill is reached is called the range and it corresponds to the distance at which the variables are no longer spatially correlated. The value that the estimator $\gamma(h)$ takes when $h=0$, is called the nugget effect, which represent a systematic error caused my micro-scale variations and/or measurement errors.

Following the calculation of the variogram, a variographic model must be fitted to it. The variographic model is an analytic expression which captures the behaviour of the spatial variability observed in the variogram. The most commonly used fitting models are the: a) linear, b) exponential, c) spherical and d) Gaussian models. The adjusted variographic model will then be used in the kriging process to estimate the studied variable at a spatial coordinate as a function of the separation distance between the research point and the sampled data.

It is not recommended to over fit the variographic model as the goal of this step is to capture the major spatial features and general behaviour of the spatial variability. When different models provide similar fits, one should select the simplest one. The more complicated model does not usually lead to more accurate estimates (Goovaerts, 1997).

3.4 *Ordinary kriging*

Kriging is a method that allows the estimation of a regionalized variable on every coordinates of a research grid by a linear combination of punctual neighbouring data. It is the best linear unbiased estimator (*BLUE*). The method is considered as unbiased because the estimation average error is null and as the best estimator because the estimation error variance is minimized.

To allow ordinary kriging, the analysed data must be stationary, namely that each observations must follow the same probability law, to have the same average and variance and that the auto-covariance between a pair of data be independent from their spatial position.

The kriging equation can be developed as a summation (Eq. 2) or as a matrix (Eq. 3).

$$\hat{\gamma}_{io} = \sum_{j=1}^{n} \omega_j \hat{\gamma}_{ij} - \mu \;\; \forall \;\; i = 1,2,\dots,n \qquad (2)$$

$$\begin{bmatrix} \hat{\gamma}_{11} & \hat{\gamma}_{12} & \cdots & \hat{\gamma}_{1n} & -1 \\ \hat{\gamma}_{21} & \hat{\gamma}_{22} & \cdots & \hat{\gamma}_{2n} & -1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \hat{\gamma}_{n1} & \hat{\gamma}_{n2} & \cdots & \hat{\gamma}_{nn} & -1 \\ 1 & 1 & \cdots & 1 & 0 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \\ \mu \end{bmatrix} = \begin{bmatrix} \hat{\gamma}_{1o} \\ \hat{\gamma}_{2o} \\ \vdots \\ \hat{\gamma}_{no} \\ 1 \end{bmatrix} \qquad (3)$$

where γ is the spatial variability estimator (Eq. 1), ω the weight of the estimation and μ the Lagrange multiplier. Kriging is therefore a system of $n + 1$ equations with $n + 1$ unknowns that can be solved by gaussian elimination. This process is to be applied for each coordinates of the interpolation grid, for which a $n + 1$ by $n + 1$ matrix will have to be solved, where n is the number of sample pairs formed between the grid coordinate and every observations within the correlation range.

The results of kriging are an estimation of the variable at a given point and its estimation error variance σ_R^2 (Eq. 4) which is a very good indicator of the estimation's relative quality.

$$\sigma_R^2 = \sum_{i=1}^{n} \omega_i \hat{\gamma}_{io} - \mu \qquad (4)$$

4 FIELD DATA ANALYSIS

4.1 *Nature of the field data*

Data used for the analysis of the dam's core are surveillance data collected during the construction of the structure. Materials put in place were regularly sampled after being compacted for laboratory testing of the granulometric curves, water content, saturation degree and density. *In-situ* tests were also realized using a nucleo-densimeter to measure water content, saturation degree, density and compaction degree. Every sample's spatial coordinates and time stamp are also available.

4.2 *Spatial interpolation grid*

The spatial and temporal coordinates available for each sampling point allows the reconstitution of the construction sequence and the ordering of the samples into a continuous sequence. For this project, each sample was placed on a 2D continuous plane that represents the core as if each lift was placed next to each other instead of on top of each other. This step is realized to insure the data is ordered in a way that respects its spatial continuity. As Venkovic et al. (2013) demonstrated, because dams are built by stacking lifts on top of each other, spatial variability only exists in the direction of the construction sequence. Even if two lift are on top of each other, it is possible that they were built with significantly different materials and should not be considered continuous and therefore evaluated in the elevation axis.

The interpolation grid used for the analysis of the dam's core is a sequence of rectangles of the same dimensions of each lifts, each of these rectangles are divided in 1m/1m cells and put next to each other following a continuous x axis corresponding to the *station* coordinate. The interpolation grids y axis corresponds to the *core width*. The resulting grid varies from 66 m to 4 m wide (y axis) and is 64 360 m long (x axis). The sample data's *station* coordinates were transformed into a continuous x axis to have the same coordinates than the interpolation grid.

4.3 *Empirical permeability calculation*

Permeability was not measured during the dam's construction and an empirical model was thus used to estimate this parameter. The model retained in this study was developed by Leroueil et al. (2002) from the analysis of a large number of permeability results of till samples generally used in the construction of compacted till embankment dams, which showed that strong relationships exists between the fines fraction (passing ≤ 80 μm) or clay fraction (passing ≤ 2 μm) of a compacted till. Fig. 4 shows the relationships with clay fraction that had the strongest correlation with permeability. Those relationships are therefore used in this study.

The model in Fig. 4 shows the influence of fabric on the relationship between the permeability and the clay-size fraction of a compacted till. As observed, the influence of the compacted till fabric on permeability is quite important as its impact can make the permeability many orders of magnitude. This difference is caused by the conditions in which a till is compacted.

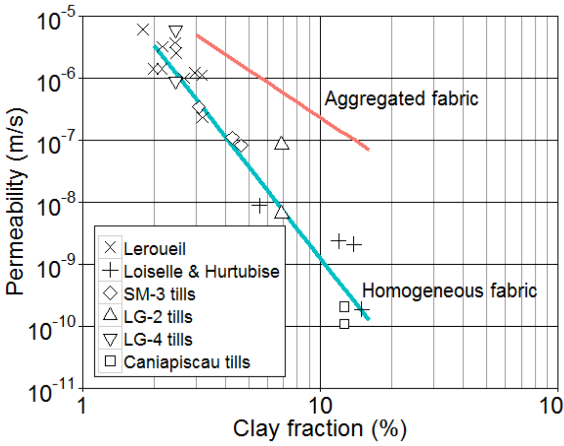

Figure 4 - Relationship between hydraulic conductivity, clay-size fraction and fabric. (Leroueil et al. 2002)

As shown in Fig. 5, if the till is compacted on the wet side of compaction curve ($S_r \geq S_{r\ opt}$), the pores sizes of the compacted till will tend to be homogeneously distributed in the material. On the other hand, if the till is compacted on the dry side of the compaction curve ($S_r < S_{r\ opt}$), the pore size distribution will be evenly distributed and associated to macro pores formed between aggregated clay particles. If the pore structure is heterogeneous, the clay particles are aggregated and the permeability is much higher than for the homogeneous structure because macro pores greatly influences permeability.

The model developed by Leroueil et al. (2002) is best suited for this study because it takes into account the fabric while typical empirical, semi empirical and theoretical models such as Kozeny-Carman are essentially based on homogeneous structure of soils and cannot account for the influence of fabric. The relationships of Fig. 4 were developed using data from northern Québec dams, including the dam studied in this project. The till properties used for the construction of the core are well within the range of the model, the model is therefore well suited for the estimation of the permeabilities in this study.

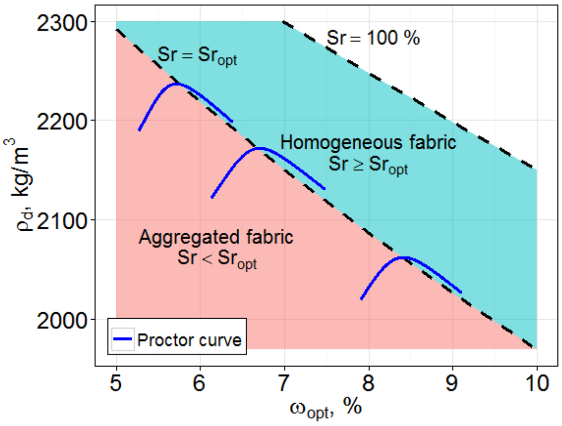

Figure 5 - Proctor compaction curve and fabric

The clay fraction – permeability relationships of Fig. 4 can be mathematically expressed by:

$$k_H = 0.0001 * P^{-4.902} \quad \text{(homogeneous)} \quad (5)$$

$$k_A = 0.00008 * P^{-2.54} \quad \text{(heterogeneous)} \quad (6)$$

where k_H and k_A are respectively the homogeneous and aggregated permeabilities (m/s) and P is the clay fraction (% by weight).

5 RESULTS

The geostatistical method was applied on the $\log_{10}$ transformed permeability data, estimated with equations (5) and (6). The statistical properties of the data set are shown in Tab. 2. The presented statistical properties in Tab. 2 focus on describing the data's distribution. As kriging assumes for continuous data, it is an optimal predictor when data follows a normal distribution, skewness, a measure of the asymmetry of the probability distribution of a random variable, is a rather interesting parameter. For the studied data, the skewness values are 1.44 and -1.19 for the untransformed and log-transformed data respectively. The rule of thumb to interpret this parameter is if the skewness is greater than 1 or less than -1, as it is the case here, the asymmetry is considered as substantial (Bulmer, 1979).

Although the transformed data still shows significant skewness, an important improvement is observed from the original data. Doing so allows for a clearer description of the spatial variability and ultimately a better variogram which better represent the spatial behaviour of the studied parameter.

Table 2 - Statistical properties of permeability

Statistical properties	k (m/s)	$\log_{10}$(k)
Number of observation	363	363
Minimum	7.34e-10	-9.13
Maximum	1.64e-5	-4.79
Average	2.65e-6	-6.07
Standard deviation	3.30e-6	0.85
Variance	1.09e-11	0.73
Skewness	1.44	-1.19

The permeability variogram is presented in Fig. 6. The spherical variographic model was used as it shown it was best fitted to model the spatial variability behaviour of the studied parameter. The nugget effect value is 0.445, the sill 0.711 and the range 12200 m.

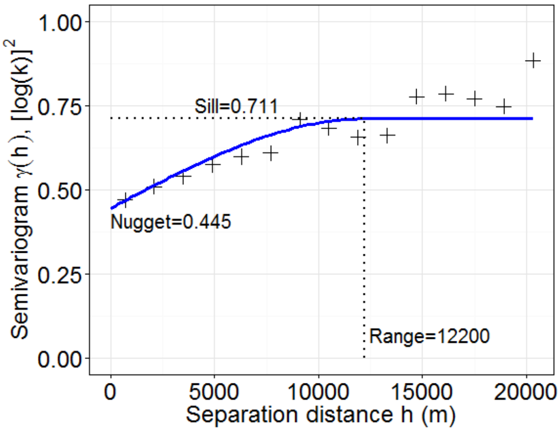

Figure 6 - Variogram, $\log_{10}$(k)

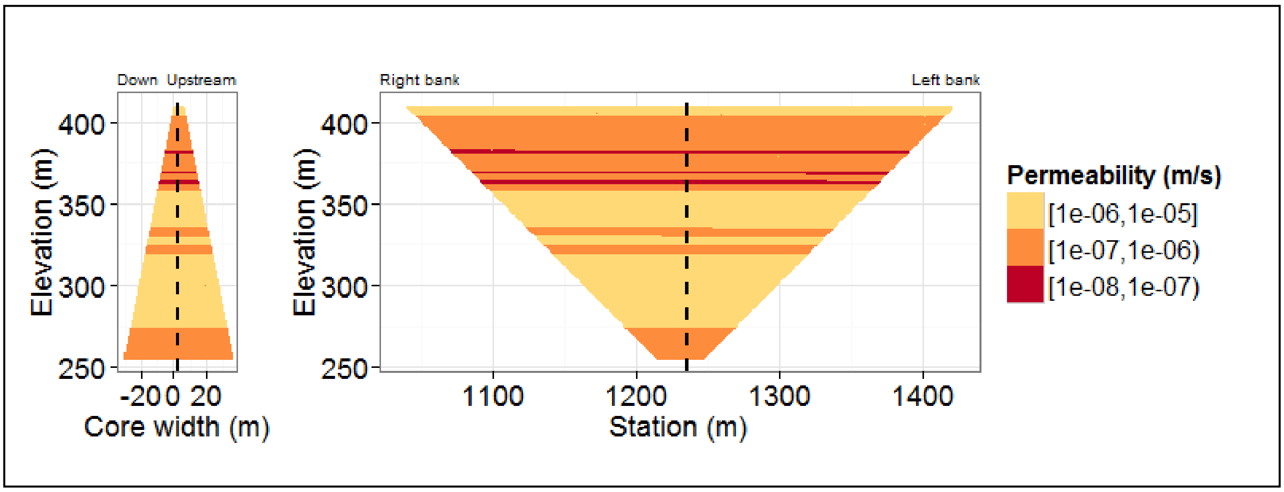

Figure 7 - Permeability ordinary kriging estimation results

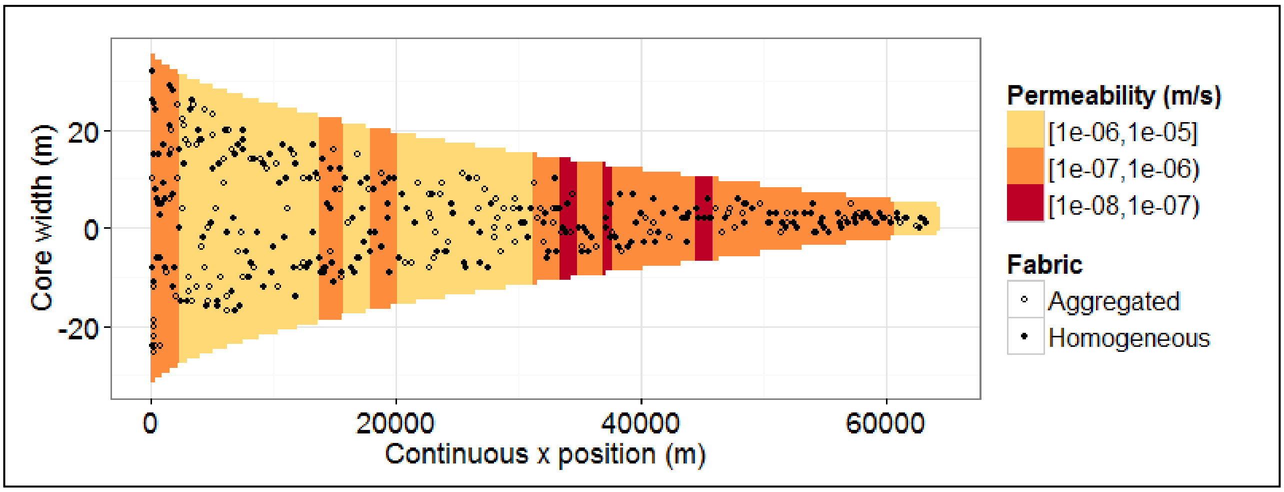

Figure 8 – Permeability ordinary kriging estimation results on unfolded core structure

Finally, Fig. 7 and 8 presents the results of ordinary kriging. Fig. 7 shows the results from two different directions: upstream-downstream and right bank-left bank and Fig. 8 shows the same results placed on the unfolded 2D plane with the fabric of sample data.

6 DISCUSSION

The stratified structure of an embankment dams compacted till core is already well known and the results from this study confirms the results from previous studies. The results shown in Fig. 7 reveal that the geotechnical properties of the core don’t solely varies according to elevation, but can also slightly fluctuate within a single lift in every axes, these variations can easily be seen around elevations 330 and 360 m.

Two areas which shows permeabilities of the order of 10^{-6} m/s located in the 360 to 330 m and 320 to 275 m elevation intervals were essentially built using tills from borrow pits 9B and 9Est, which have average clay-fraction values of 3 and 2.8 %. According to Fig. 4, for these clay fractions value, average homogeneous permeabilities should be expected to range from $4.6 \cdot 10^{-7}$ to $6.4 \cdot 10^{-7}$ m/s. Except that based on the optimal and *in-situ* saturation degree fabric criterion, most of the tills from borrow pits 9A and 9Est were compacted on the dry side of the optimal and therefore display an aggregated fabric, which results in higher average permeabilities ($1.7 \cdot 10^{-6}$ and $1.6 \cdot 10^{-6}$ m/s respectively), as observed in Fig. 7 and 8.

Another interesting feature is the bands that shows permeabilities of the order of 10^{-8} m/s around elevations 375 m which corresponds to areas where tills from borrow pit 12 were used, which showed a significantly higher clay-fraction. It is also observed in Fig. 8 that most samples from this area have a homogeneous fabric. Both the higher clay content and homogeneous fabric explains the lower permeability observed in this area. The areas where permeabilities of the order of 10^{-7} m/s are observed corresponds to homogeneous fabric compacted tills from borrow pits 9, 9A, 9B and 9Est or aggregated tills from borrow pit 12.

According to this ordinary kriging estimation, 57% of the volume of the core is occupied by compacted tills of permeability 10^{-6} m/s or higher, 39% of permeability of the order of 10^{-7} m/s and 4% of permeability 10^{-8} m/s or lower.

The relationship between permeability, clay fraction and fabric used for this study highlights some of the vulnerabilities of compacted till cores. For this case, a low clay fraction materials (9, 9A, 9B, 9Est) and a high clay fraction materials (12) have been used for the construction. The average permeability of a low clay fraction material with a homogeneous fabric is of the order of 10^{-7} m/s, while the same material with an aggregated fabric shows permeabilities of the order of 10^{-6} m/s. The same difference in the variation in the order of magnitude is observed for the high clay fraction material, for which the homogeneous and aggregated permeabilities are of the order of 10^{-8} m/s and 10^{-7} m/s respectively. This relationship shows that the higher the clay fraction is the more important will be the impact of the fabric variations. If a high clay fraction material is put in place with an aggregated fabric, its permeability will be around the same range as a homogeneous low clay fraction material.

The use of geostatistics allowed the assessment of the dam’s core geotechnical properties for every 0.5 m^3 of the structure, a feat that couldn’t have been possible without the use of powerful mathematical tools. An analysis of those properties on such a high resolution allows a better understanding of the variability and distribution of the materials properties, to target areas which might prove problematic and to better anticipate the problems which might arise during the operational phase of the dam. Coupled with different models, geostatistics could be used to precisely measure seepage rate, predict the behavior of the dam under specific constrains, thus making it a useful tool in the context of climate changes for example, or even to predict the effect of aging on the structure, using time based monitoring data.

However, the precision needed for such modeling is not yet achieved. Different factors could be addressed in order to refine the quality of the estimation. For instance, the trend removal applied in this study could benefit from a finer approach. The use of tills from borrow pit 12 proved problematic for the geostatistical analysis as they introduced two areas which showed very different parameters than the rest of the structure. Such a discrepancy is sure to affect the overall results as kriging is strongly affected by extreme data. Possible solutions to counter that problem could lie in using a) a kriging approach based on different domains, b) universal kriging or c) co-kriging. Another possible avenue to improve the present approach would be to first model each parameter linked to permeability such as clay fraction and fabric separately or as a multi-parameter system and then use the resulting estimations of these parameters in the permeability relationships (Eq. 5 and 6). This last approach sounds very promising as it allows the assessment of the spatial variability behavior of each parameter, which can be very different from each other.

7 CONCLUSION

The stratified structure of dams was already suggested by past use of geostatistics to study the geotechnical properties of compacted till core embankment dams. The use of a 2D plane and spatially correlated data showed the same behaviour and also highlighted that spatial variability occurs not only according to elevation, but in every axes of the core’s structure.

Geostatistics and kriging proved a very interesting modelling tool for the assessment of dams’ geotechnical properties and in this case permeability. It was shown that the permeability of the studied dam core can vary from 10^{-6} to 10^{-8} m/s accordingly to the source of the till used in the various area of the structure and construction conditions.

Using an empirical relation including a fabric parameter proved very useful in the analysis of core's permeability, a structure known for its vulnerability to variations of its compaction conditions. The influence of fabric on permeability was indeed very important and the results of ordinary kriging estimation showed the high level of correlation between the two parameters.

The goal of this study was to show that it is possible to use geostatistics and kriging as a reliable tool of modelling dams. Using construction control data with geostatistics is a great way to valorize otherwise less used data and allows the modelling of the dam even before the impoundment, thereby having a very interesting way of predicting the dam's behaviour before it is actually in use.

The approach used in this study can still be greatly improved, especially by refining the kriging strategy of permeability parameters and samples distribution. By improving the approach, the obtained estimations will be more reliable and more precise, which will allow for more diversified uses.

8 ACKNOWLEDGEMENTS

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