Modelling of Earth Pressure from nearby Strip Footings on a Free & Anchored Sheet Pile Wall

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ABSTRACT
A strip footing from a nearby civil or industrial building or railway track is frequently situated near a sheet pile wall. Assessment of the extra pressure on the wall generated by the footing causes theoretical problems for the designer. The distribution of this pressure depends in fact on many parameters. Besides the location and magnitude of the load, a characterization of the soil and the wall is necessary for a rational design. Furthermore, the movement of the wall has a significant impact on the pressure. In this paper, both a free and an anchored wall are investigated. The problem is solved by means of different analytical methods compared with finite element modelling applied to a number of representative load cases. These comprise different strengths for the cohesionless soil and different load scenarios. After the study of a number of existing methods, simple and robust solutions are proposed for the future design of the sheet pile walls.

Keywords: Sheet pile wall, free wall, anchored wall, strip footing, earth pressure, additional pressure, finite element method, sand, stress distribution

1 INTRODUCTION

Driven sheet pile walls play an important role in many ways, both to overcome topological differences and in connection with excavations often near existing buildings. When the wall is driven in cohesionless soil a robust design is necessary to maintain the integrity. Generally, a substantial resistance against bending is required in the sheet pile wall to resist the pressure on the backside of the wall. This pressure can be a pressure from a water table on the backside, the earth pressure from the self-weight of the soil, and a load on the ground surface behind the wall. The load on the surface may arise from foundations of nearby buildings or from trafficking.

The influence on the wall pressure from especially shallow footings is often difficult to assess in practice and crude estimates are often used in lieu of methods that are more precise.

The paper, which is a continuation of authors work in Denver & Kellezi (2011) & (2013), describes methods to calculate more accurately the additional earth pressure on the wall from a strip or continuous footing behind the wall. Different aspects in connection with loads behind the walls are mentioned and discussed.

A free (unanchored) wall, where the top of the wall moves toward the excavation during rupture, and a wall rotating clockwise about an anchor, (the tip moves against the excavation wall), are investigated. The results can be used in connection with any method to calculate the ordinary wall pressure. However, an effort has been made to integrate the problem into the Danish method of sheet pile wall design. The method is outlined in the following, illustrating the problem within this frame. The reason for this is that the method is based on actual rupture figures in the soil with respect to the predicted movements of the wall.

2 DANISH EARTH PRESSURE CALCULATION

The Danish earth pressure calculation (EPC) has been introduced by J. B. Hansen (1953) and used in Denmark for half a century. In this method the principle of superposition is used as shown in equation (1) for the normal stress on the wall. Here *K* is the earth pressure coefficient (different for the three terms). The first term represents the pressure from the selfweight of the soil, γ' is the effective unit weight of the soil and *z* is the depth along the wall to the point investigated from the soil surface. The second term is the contribution from an infinite surface load (*p* or *q*) on the soil surface behind the wall. The third term is the contribution from a cohesion (*c*). In this paper no cohesion is assumed.

$$e(z) = \gamma' z K_\gamma + pK_p + cK_c \qquad (1)$$

The water pressure (if any) is finally added to find the total pressure.

In the Danish method the wall is considered composed of several rigid parts interconnected by yield hinges. Each part is assumed to rotate about a point and the earth pressure coefficients are functions of the position of this point and the direction of rotation (besides the friction angle of the soil, φ). A few examples of rupture figures used for calculation of *K* are shown in Fig 1. Examples of walls with yield hinges are shown in Fig. 2.

The result of each calculation, is the total force on the wall and the point of application. The normal component of this force (*E*) is applied on the wall in a way to obtain a safe design. E.g. when the upper part of a wall, (above an anchor level), moves against the soil in failure, a large part of *E* is applied near the top, corresponding to a passive Prandtl rupture zone. A pressure jump near the top is thus assumed to ensure that the effect of the distribution, (in terms of total force and moment), is equal with the results from the calculations of the rupture figure.

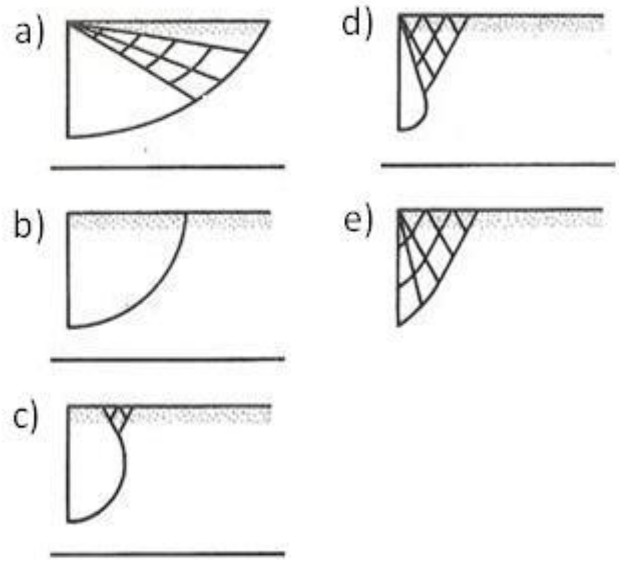

Figure 1 Rupture figures with different rotation points for a stiff wall. Type (e) (with rotation point near the tip) is used for a free wall and type (b) is used for an anchored wall.

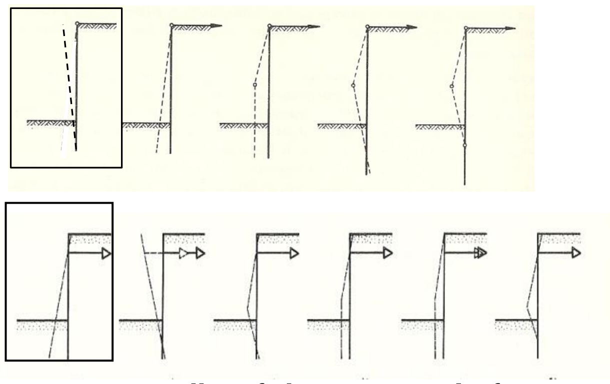

Figure 2 A wall in failure composed of one or more rigid segments connected by yield hinges in failure. This paper deals with the two left hand cases

The method has been described by Mortensen & Steenfelt (2001) and results of different examples calculated, are here compared with finite element (FE) two-dimensional (2D) analyses.

3 DESIGN PROGRAM "SPOOKS"

Although J. B. Hansen (1953) has made a complete set of diagrams to find the values of *K*, the earth pressure calculation for a specific design situation is rather time consuming. To this end Geo, the previous Danish Geotechnical Institute, has made a commercially available computer program called WINSPOOKS to overcome this problem.

Here, apart from the geometry of the excavation, the soil conditions and water tables, only a selection of the total wall movements, as shown in Fig. 2, is necessary as input. The results are a distribution of both, earth and water pressures, diagram of bending moments along the wall, tip level, and anchor force (if any). Altogether, ready for the final selection of the sheet pile profile and anchor.

The scenario when a surface load is present, starting at a certain distance from the top of the wall, can be calculated in WINSPOOKS. This is true if the load is active at an infinite width, which means that *b* in Fig. 3 included in section 5, continuous to infinity. This is incorporated by applying the full surface load at a certain depth below the soil surface. However, if b is finite, the effect of the load on the wall can´t be estimated by WINSPOOKS. In this case, the extra wall pressure must be assessed differently and inserter manually into the program.

4 PARTLY LOADED SURFACE

Applying the principle of superposition, the additional pressure from the strip load can be calculated separately and added to the total pressure on the wall, as a second term. This term is rather complicated to assess. The parameters are *a*, *b* as shown in Fig 3, γ', *p*, and *z*, beside the movements of the wall as referring to Fig 2.

The total number of parameters can be reduced if the problem is treated in a dimensionless form. Still, too many parameters remain to derive a general complete solution applicable to engineering practice. This means that the problem can only be solved by choosing a number of typical cases, calculating them conventionally and numerically. By comparing the results, a simple solution can be derived, to be used as a reasonable approximation in an actual design situation. In the following, different approaches will be discussed.

5 COULOMB'S EXTREME METHOD

An extreme method was early presented by Coulomb (1776). The principle is that straight rupture lines are used to confine a rigid sliding body. This method can be used to calculate the influence of a partial surface loading on a wall. The method will be outlined in the following as it is a serious candidate to a solution of the problem. In Fig. 3 the method is outlined for the present problem.

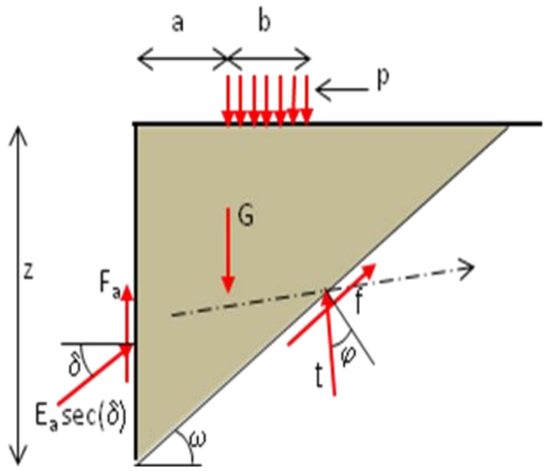

Figure 3 Coulomb's Method

The geometry appears from the figure and *G* is the weight of the shaded body, *t* is the total force from friction on the rupture line, and *f* is the shear force from the cohesion if any. The outer support on the wall consists of a normal force E_a and a shear force F_a. The latter comprises the effect of a wall adhesion (a_h). The frictional roughness is described by a wall friction angle (δ). As the problem is 2D all forces (single arrows in the figure) have units of force / length, whereas the distributed load p has the dimension force / length^2.

The principle is now that the forces and the load are projected on a line perpendicular to t (the stipulated arrow) and equilibrium is required. This means that the value of the unknown t vanishes. With a given value of ω the force E_a can be determined as $E_a(\omega)$. The value of ω is now varied and max$E_a(\omega)$ found as the necessary pressure to maintain equilibrium. The figure is made corresponding to a sliding movement to the left. This means that the results correspond to the active pressure. If this procedure is repeated for different values of z, the pressure distribution can be found as $e(z) = \mathrm{d}E / \mathrm{d}z$ and only applied when e is positive. The rupture line may not meet the soil surface in the so-called correct angle (i.e. it is not possible to construct a Mohr's circle for this point). For this reason, the static conditions are not generally fulfilled for the solution. Furthermore, the straight rupture line is in most cases a crude approximation to the often far more complex boundary rupture line for a more correct rupture figure in Fig. 1.

It is a Danish experience that reasonable solutions are found for wall problems with active ruptures, whereas unusable solutions are found for soil in passive rupture. In this paper, this method has only been applied for walls with soil in active rupture.

6 THEORY OF PLASTICITY

A method to assess the extra soil pressure caused by a partial load on an anchored wall has been introduced by Steenfelt and Hansen (1984). The Danish method to calculate the earth pressure coefficient from a relevant rupture line has been adopted. A circular rupture line is used as an appropriate choice for a rotation about a point at the anchor level. The stresses from the rupture line are determined by the Kötter's differential equation. The total force is found by integration of this equation and presented by Hansen (1953) and shown as the resulting force (F_o) and moment (M_o) about the centre of the circle as shown in Fig. 4.

It should be mentioned that the stress in the starting point of the integration (the top of the rupture line) is assessed empirically as no complete equilibrium can be achieved here.

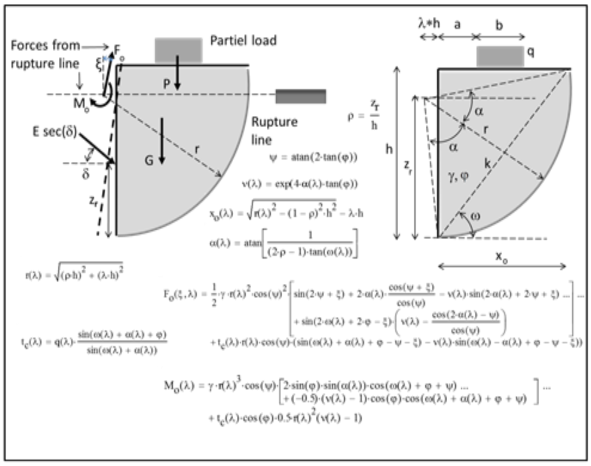

Figure 4 An analytical method where a circular rupture figure is applied. Negative values of φ and δ shall be applied as the rupture is active. The almost eligible formulas are only included to illustrate the complexity of the method.

On the basis of integration of the forces along the rupture line the optimal circle can be determined and the total pressure on the wall calculated. The method is introduced and discussed in details by the authors and the results of a large number of load scenarios are presented in their paper. The authors have made a computer program to solve the problem by the described method. However, some theoretical problems exists when the rupture line starts near or under the loaded area. Furthermore, it can be strongly disputed if the Kötter´s equation in fact can be used when the soil is only partly loaded. Consequently, results from calculations are not included in the final comparison.

7 EMPIRICAL METHOD

It is usual practice to partly apply a soil pressure derived from the distribution of the uniformly loaded surface, where the load itself is multiplied with a factor. A minor part of this distribution load, multiplied with another factor, is applied on the wall in a depth interval confined by in inclined lines from the loaded area through the soil.

In Fig. 5, a method of this kind, often used in Denmark, is shown.

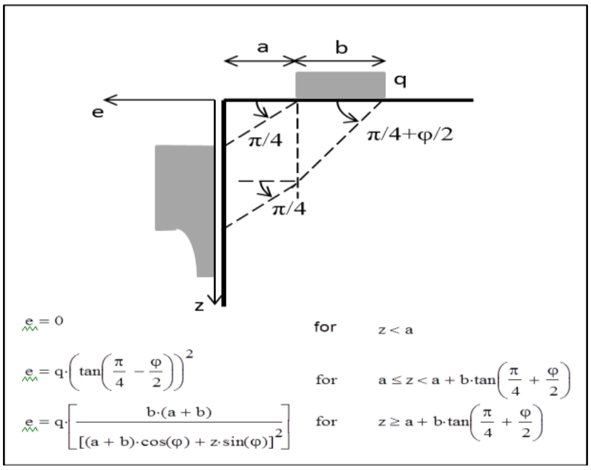

Figure 5 An empirical method partly based on the Coulomb's earth pressure theory

However, a tail below the lower line has been proposed by Mortensen (1976). The author has also pointed out the complexity of the problem and the assumption is a smooth wall that rotates anti-clockwise about a point below the tip of the wall. Consequently, the upper part with the even distribution is given by an active Rankine rupture figure. The tail is probably inspired by calculations by Coulomb's method where the lower part is more dependent of other parameters than *a* and *b*.

This solution has been applied for comparisons regarding the free walls with soil in active rupture for which it is derived.

8 ELASTIC SOLUTION

An elastic solution developed by Boussinesq (1885) as referring to Fig. 6 is also often used because of its simplicity. Besides the theory of elasticity a smooth vertical wall, without any movement, is assumed. This method is often questioned as the resulting distribution is expected to be inaccurate due to the fact that the wall in fact moves during rupture. This is also the authors experience when the movement of the wall is anti-clockwise about a low point in the wall. However, if the movement is a clockwise rotation about the anchor installation point, the assumptions for an elastic solution are more relevant. Consequently, this method has been included in the comparison for anchored walls.

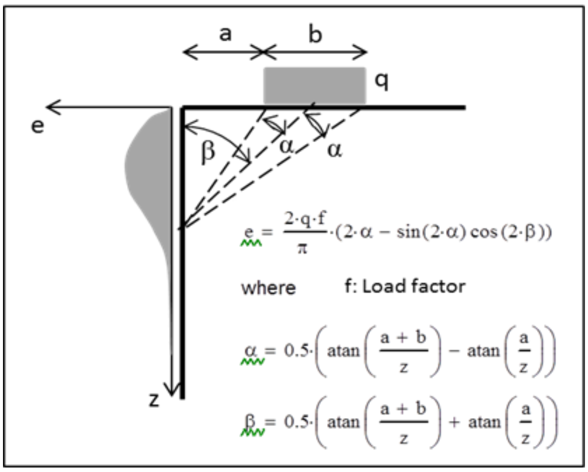

Figure 6 Elastic solution by Boussinesq (1885)

An appropriate triangular distribution as referring to Fig. 7, which approximates the elastic solution, is often used in Denmark because of its simplicity. This approximation has been used in the comparison.

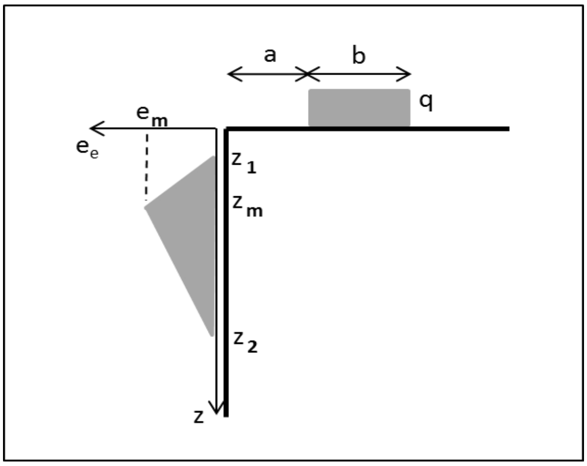

Figure 7 Triangular approximated distribution for the Boussinesq´s solution where the strip footing is assumed as a line load (z_1 = 0; z_m = 0.4(a+0.5b); z_2 = 2.5(a+0.5b); e_m = 0.45qb/(a+0.5b).p.

9 2D FE PLANE STRAIN MODELLING

In order to evaluate and rank the different conventional methods, a number of load scenarios have been calculated and analysed by the FE program Plaxis (2012). A 2D FE mesh has been generated using triangular finite elements (15-noded). Sand is modelled in drained conditions using Mohr-Coulomb constitutive model. Clay, below the excavation level, is modelled in undrained condition using Tresca constitutive model. This layer has been included to ensure the correct movement of the wall. The wall is modelled as weightless and rigid body. The model is constructed in such a way that the active pressure on the wall does not interact with the passive one. The initial geostatic conditions are calculated first. Mesh sensitivity analyses have been carried out and an optimal mesh pattern with respect to element size and obtained accuracy has been chosen for the final analyses.

For the free wall some results from the calculations are shown in Fig. 8, and for the anchored wall, similar results are shown in Fig. 9.

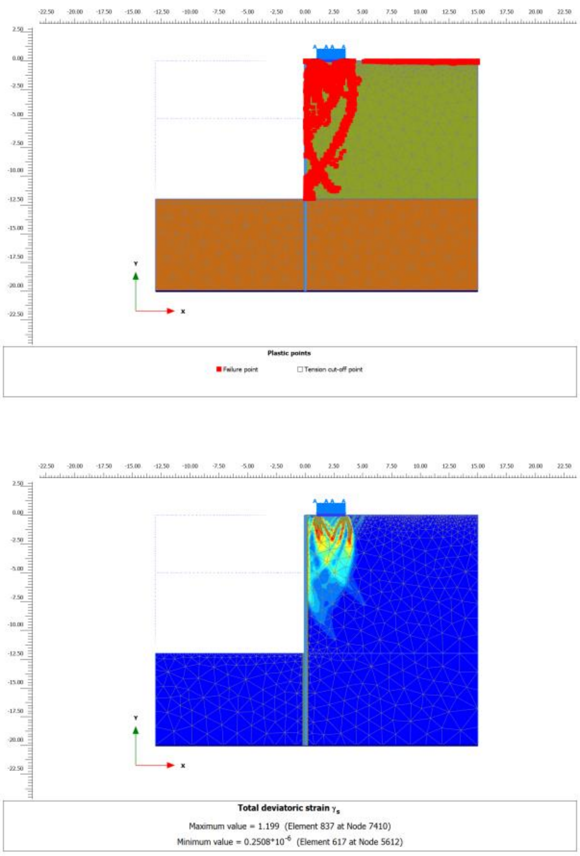

Figure 8 Free Wall, 2D FE plain strain results (plastic points & deviatoric strains), φ=30◦, a=1.0 m, b=2.5 m, or a / b=0.4, p=125 kPa.

Plaxis plastic analyses (small deformation theory) and Updated Mesh (large deformation theory) are both considered in order to see the impact the deformation / movement of the wall has on the results. The calculations are carried out in different ways considering the impact the staged construction (excavating after, before, or at the same time with the load application) has on the results.

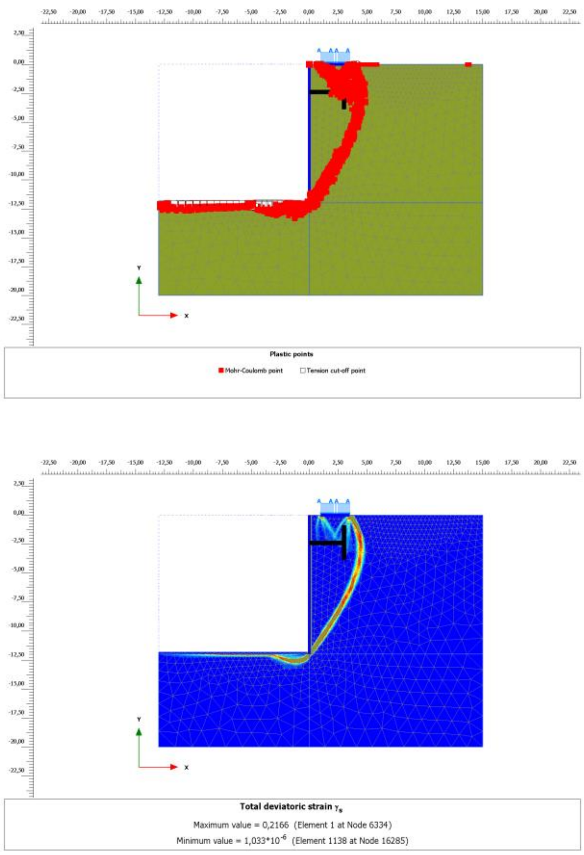

Figure 9 Anchored Wall, 2D FE plain strain results (plastic points & deviatoric strains), φ= 30◦, a=1.0 m, b=2.5 m, or a / b=0.4, p=125 kPa.

The extra pressure on the wall has been calculated as the difference between the pressure from both the soil and the strip footing or partial surface load, and the pressure only from the soil (i.e. two different calculations). From a conceptual point of view, no error is introduced by this procedure. The calculated difference can afterwards be added to the pressure from the soil alone (calculated by other conventional methods) to obtain the combined effect.

Actually, in a FE context, two different rupture patterns are subtracted. And thus, in principle 'two different degrees of total rupture' are subtracted. However, a study of the resulting pressure distribution reveals that the extra pressure is by far and large confined to the upper part of the wall. This means that similar pressure is calculated for the lower part of the wall. This is used as an argument that no substantial error is introduced by this approach.

The failure patterns given in Fig. 8 & 9 in terms of plastic points and total deviatoric strains, indicate the difference in the failure mechanism for the free and the anchored walls, respectively.

10 PROPOSED METHOD

A new method is proposed based on the overall results of the conventional and FE calculations. It is intended to derive a simple and easy to use method, which means that a simple shape of the resulting additional pressure distribution is chosen. This is in line with the recognition of the large inherited uncertainty in the determination of the distribution by simple means. The triangular distribution shown in Fig. 7 fulfils this requirement. The determining values are shown in Table 1.

Table 1 Values of proposed, triangular stress distribution behind the wall referring to Fig. 7, and with ϕ less than $\pi / 4$

Wall	Free	Anchored
z_1	$0.25\ a$	$(9a + 15b)$ $(1\text{-}\tan(\phi))^4$
z_m	$z_1 + 0.5\ a$	$z_1 + 0.5\ (a+b)$
z_2	$z_m + 6.0\ b$	$z_m + 8.0\ b$
e_m	$0.30\ q\ (b/a)^{0.4}$ $(\sin(\phi)+0.5)^{-2.5}$	$0.3\ q\ (b/a)^{0.5}$

The derived procedure of assessing the influence of a strip footing, or a partial loaded surface on a sheet pile wall, should fulfil the condition of converging to the additional load distribution usually applied for a fully loaded surface. In order to achieve that, the following procedure is proposed:

- Calculate the elastic distribution ($e_e(z)$) using the above mentioned guidelines.
- Calculate the distribution usually used for a fully loaded soil surface. Use only the part of this distribution corresponding to the interval of the uniform part of the distribution ($e_p(z)$) shown in Fig. 5.
- The final distribution is: $e(z) = W{*}e_p + (1\text{-}W){*}e_e(z)$, where W is a weight function $W = F^3$ and $F = 1.2{*}b/h$. If F>1.0 then F = 1.0 is used.

11 COMPARISON OF METHODS

The different methods, conventional and FE, yield significantly different results. In fact, the correct solution depends on many other parameters as earlier mentioned. In order to perform a meaningful analysis of the different methods, the following strategy has been applied without further discussion:

Table 2 Load cases investigated

No	φ	a	b	q	No	φ	a	b	q
	(deg)	(m)	(m)	(kPa)		(deg)	(m)	(m)	(kPa)
1	30	1	2.5	125	5	40	1	2.5	713
2	30	1	1	50	6	40	1	1	285
3	30	2.5	1	50	7	40	2.5	1	285
4	30	5	1	50	8	40	5	1	285
h = 12 m		γ = 14 kN/m³			c = 0 kPa rough wall				
Height to rotation point for anchored wall:								h_p = 9.6 m	

A number of relevant load cases has been selected as referring to Table 2. It should be emphasized that the local bearing capacity of the soil under the partial load or strip footing, is first controlled and ensured. The wall will somehow confine the rupture figure developed under the load as shown in Fig. 8 & 9. The ratio between the applied load and the unit weight has some influence on the solution though. This ratio is defined as $N = 2p\ /\ (\gamma\ b)$. With this definition N resembles N_γ from the bearing capacity formula. When choosing the different load scenarios modeled by FE, the N values were pre-calculated ensuring that the load scenarios corresponded to the same N value and bearing capacity of the footing was satisfied.

This was verified by the FE analyses where the loads were applied over a weightless rigid plate modelling the strip footing.

- The results from the 2D plane strain FE calculations are assumed to be superior to other methods. In order to evaluate the different methods, typical values, in connection with the design of a sheet pile wall, have been calculated.
    - For an anchored wall: Moments of the normal stress distribution at depths of 2.4 m (M1: near the anchor) and at the interval (4 - 8) m (M2: near an encastre point) and the shear force in the wall at a depth of 5 m. (T: to simulate the extra anchor force from the surface load).
    - For a free wall: Moment at a depth of 9 m (M2) and the transversal force (T) at the same depth, both near an encastre point.

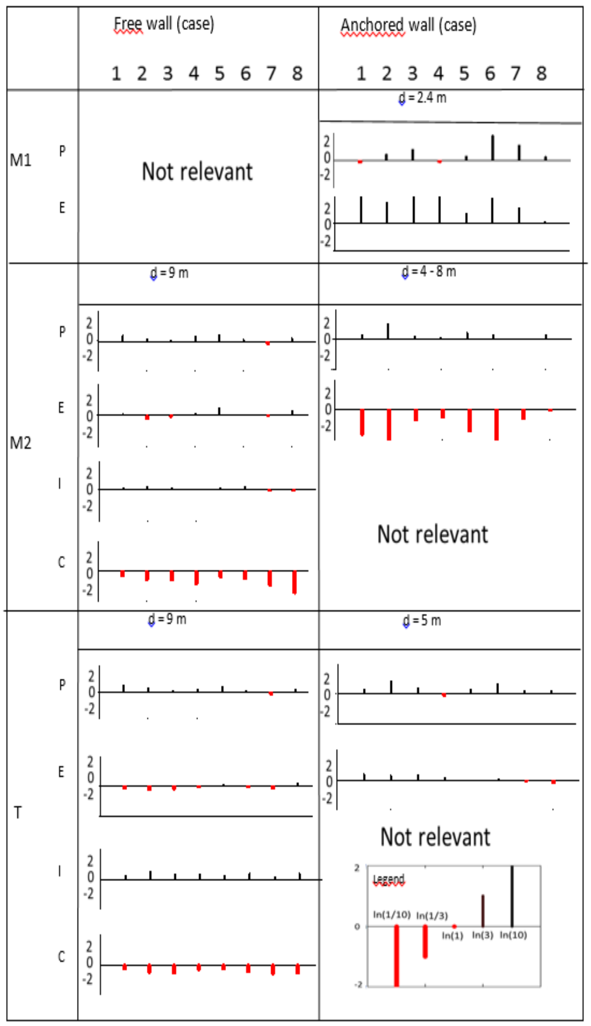

Figure 10

Figure 10 Comparison of different methods. Bars are ln(method / FE) referring to legend. M1, M2, T in the. text. P: Proposed; E: Elastic; I Empirical; C: Coulomb

- For each relevant method and each load case, the value of: ln(result for the method / FE result) has been calculated.
- The results are summarised in Fig 10.

The target values correspond to a value equal to zero (no bar). A black bar equals to +1 means that the method yields an ap. 3 times too safe value compared with the FE result. A black with height +2 (the maximum value shown) means ap.10 times or more too safe values. Values <0 are shown in red colour and mean unsafe values following the same methodology (values 1/3 resp. 1/10).

It is readily observed that:

(i) The proposed method yields superior results,

(ii) The Coulomb´s method is on the unsafe side,

(iii) The theory of elasticity yields unusable results for an anchored wall, and the empirical method yields usable results for a free wall but a calarge underestimate for an anchored wall (not shown, as the method is not intended for anchored walls).

Illustrations of the above comparison of different methods, is given in Fig. 11 & 12.

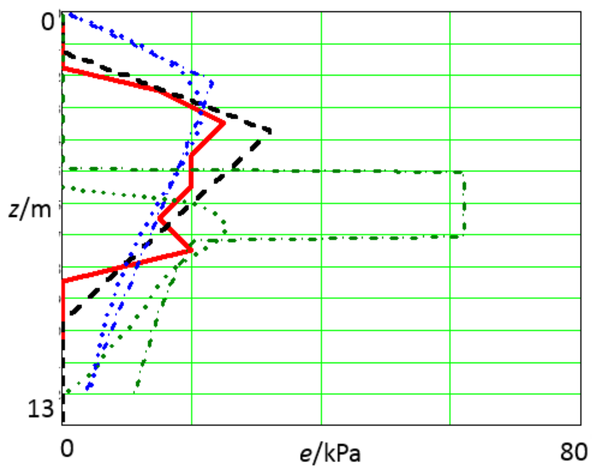

Figure 11: Example of pressure on a free wall Case 8 from Table 2. Traces from top: E (theoretical and approximation), P, FE, I, C (ref. Fig. 10).

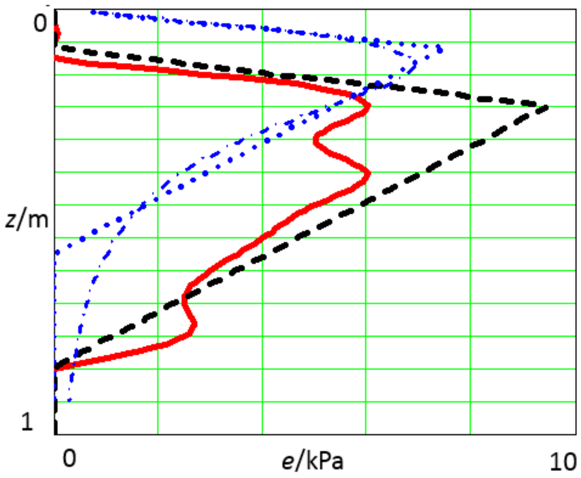

Figure 12: Example of pressure on a free wall Case 8. Traces from top: E (theoretical and approximation), P, FE, I, C (ref. Fig. 10)

The extra pressure on the wall has been calculated as the difference between the pressure from both the soil and the partial surface load and the pressure only from the soil (i.e. two different calculations). From a conceptual point of view no error is introduced by this procedure. The calculated difference can afterwards be added the pressure from the soil alone (calculated by other means) to obtain the combined effect. The only focus on the calculation is the stress distribution on the wall $e = e(z)$. If we denote the result of the FE-calculation, for both, soil and partial load, with $e_{\gamma+p}$ and the FE-calculation for soil alone with e_γ, then the extra pressure is calculated by $e_p = e_{\gamma+p} - e_\gamma$. We are now satisfied by the accuracy of each of the two terms on the right hand side of this equation, as they emerge from FE calculation, routinely use in the design situations. An extra uncertainty is of course introduced by the subtraction. But this is cancelled out when $e_\gamma + e_p$ is used in the design situation. It should be mentioned that the FE calculated e_p is reasonably comparable with the corresponding analytic calculation of this stress distribution used routinely in Denmark.

12 CONCLUSIONS

A new method is proposed to calculate the additional pressure on a free and anchored wall, respectively from strip footing or partial load next to the wall. The comparison study given in Fig. 10 clearly shows that the proposed method is superior to the others and is recommended in a design situation where the load case is reasonably comparable with the cases investigated. It should be added though that, the results depend to a large extent, on the number of other parameters, even including the design practice of the wall itself. The method can be applied, in combination with a conventional sheet pile wall design program like WINSPOOKS, for values of parameters reasonably covered by the current calculations. The method can be applied also for multi-layered soil profiles and multi-strip footings, or multi railway trucks (initial design), to be combined, verified and optimised though, by 2D FE modelling.

13 ACKNOWLEDGEMENTS

The authors acknowledge Geo for the financial support of the project. Furthermore the authors are indebted to Søren Gundorph for a fruitful discussion about the subject.

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