

Mechanical weathering effect on tailings

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ABSTRACT

Over the last century the tailing volume generation has grown dramatically due to the mineral demand. Nowadays the mining industry is producing every year millions of tons of tailings. The storage of the tailings has become a challenge due to the increased storage capacity demanded. Physical risk associated to the tailings dams is the stability itself since tailing dams are considered a walk-away solution. Physical changes as breakage and shape occur to the tailing particles affecting the stability of the fills by reduced strength properties. In order to understand the reduction and shape changes of tailing particles degradation test by milling attrition (erosion) and image analysis was conducted. Uniform fractions 1-0.5, 0.5-0.25, 0.25-0.125 and 0.125-0.063mm were used.

Results have shown that attrition agents e.g. ball attrition can increase the physical erosion but also change the shape of the particles compared with autogenous attrition. However particles shape has become more regular (less elongated) and rounded in coarse fractions 1-0.5 and 0.5-0.25mm while smaller fractions 0.25-0.125 and 0.125-0.063mm seems to have opposite behavior. Comparison with previous milling studies show consistent differences probably due to the breakage of the particles was the objective. In perspective if tailings become more rounded the strength could be compromised. More studies are needed to verify this.

Keywords: Particle shape, particle size, attrition, tailings.

1 INTRODUCTION

Tailings are the mining industry leftovers, for its storage tailing dams are constructed. In the history of the mineral extraction tailings dam incidents has occurred e.g. Val di Stava tailings dam (1985) in Italy and Cananea tailings dam (2014) in Mexico.

As a consequence of the operation and raising procedures of tailings dams, the conditions in the tailings dams could be considered to be dynamic in a longer perspective. Grain size distribution, formation of layers, pore pressures and stress states are continuously changing during the operation (Ormann, et al., 2013).

In general particle degradation occurs due the environmental factors as chemical reactions (Kossoff, et al., 2012) and mechanical factors as stresses generated due the load and creep (Valdes & Koprulu, 2007). Tailings are also affected in the same way.

Mechanical behaviors of granular materials are affected by characteristics as particle size (Islam, et al., 2011), particle shape (Santamarina & Cho, 2004), size distribution (Cabalar, 2011), mineralogy (Clayton, et al., 2004) among others. Erosion changes all physical characteristics thus, its strength and behavior changes through the time.

Tailing dams need to be built with environmental and structural safety. They must stand for long periods in a long time

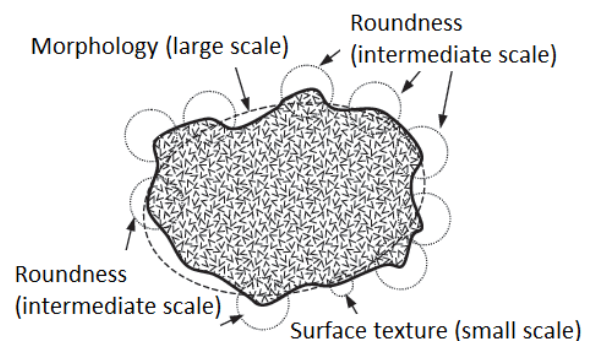
perspective, even after mining reclamation. To ensure the structural safety not only actual soil characteristics must be taken in consideration but also further changes. Laboratory mill attrition by uniform sized particles was performed in this study to determine the size and shape changes. Shape according to Cho, et al. (2006) influence the strength of the materials and, from this point of view if particles become more rounded with the pass of time due to mechanical wearing the stability could be compromised. Results have shown that ball attrition is more effective to erode the tailing particles, ball attrition produce more size reduction (as it was expected) and more shape changes. Results in this study also indicate that coarse fraction 0.5 and 0.25mm become more rounded but smaller sizes are more angular. The effect of the size reduction as the contradictory shape changes effects should be studied in in detail in further research. The statistical methods Mann-Withney test and two sample t-test used to determine shape differences have 96% of agreement, furthermore there is no skew evidence over the results on any of the methods. Thus, it is possible to use two samples t-test for the study of the tailings.

2 THE PARTICLE SHAPE

In this study the word *shape* is used to describe a grain's overall geometry. Furthermore, in order to describe the particle shape in more detail, there are a number of terms, quantities and definitions used in the literature. Some authors (Mitchell and Soga, 2005 and Arasan, et al., 2010) are using three sub-quantities describing the shape but at different scales. The sub-quantities are *morphology/form*, *roundness* and *surface texture*. In Figure 1 it is shown how the scale terms are defined.

At large scale a particle's diameters in different directions are considered. At this scale, describing terms as *spherical*, *circular*, *platy*, *elongated* etc., are used. An often seen quantity for shape description at large scale is *sphericity*. Graphically the considered type of shape is marked with the dashed line in Figure 1. At intermediate scale is focused on

description of the presence of irregularities. Depending on at what scale an analysis is done; corners and edges of different sizes are identified. By doing analysis inside circles defined along the particle's boundary, deviations are found and valuated. The mentioned circles are shown in Figure 1 **Error! Reference source not found.**; a generally accepted quantity for this scale is *roundness* or the antonym: *angularity*. Regarding the smallest scale, terms like *rough* or *smooth* are used. The descriptor is considering the same kind of analysis as the one described above, but is applied within smaller circles, i.e. at a smaller scale. *Surface texture* is often used to name the actual



quantity.

Figure 1 Particle describing the shape scale attributes (Mitchell and Soga, 2005).

3 METHODOLOGY

In this study samples from the Aitik mine has been used. The Aitik tailing dam is located about 100 km North of the Arctic Circle in the boreal parts of Northern Sweden about 15 km from the community of Gällivare. The value mineral is chalcopyrite (CuFeS_2). Main sulphides are pyrite, chalcopyrite and sphalerite. Main gangue minerals are quartz, feldspar, plagioclase and mica (Lindvall, M. and Eriksson, N., 2003).

Four range sizes were used in the actual research (see Table 1). Further on the lower size in the range will be used for convenience. Wet sieving was used with Sodium Diphosphate decahydrate ($\text{Na}_4\text{P}_2\text{O}_7 \cdot \text{H}_2\text{O}$) as a dispersant to enhance the particle separation. After sieving specimens were dried for 24 hours at 105°C .

Smooth steel drum mill with 115mm in diameter and 132mm in length over a rolling

table was used to generate a gentle and a constant flow of tailings material over a constantly rebuilding slope. Speed of 60 rpm (revolutions per minute) was used to ensure a gentle rolling of the particles down the slope and procure the attrition/wearing/abrasion of particles. Speeds close to the terminal velocity were avoided due to it could breakage the particles.

Table 1 Test specimens and particle size ranges

Specimen	Range (mm)
0.5 _x	1-0.5
0.25 _x	0.5-0.25
0.125 _x	0.25-0.125
0.063 _x	0.125-0.063

The tests were configured in two different forms the first was using balls to speed up the attrition of the tailing particles and the second was autogenous attrition (no balls). The subscript “x” in Table 1 represents the autogenous attrition (absence of balls) for “a” and the use of balls for “b”. A total of 200 gr of tailings were used during each batch with 1000 ml of water. For ball attrition 100 grams of 7 millimeters in diameter steel balls were included.

Degradation was conducted for 2 and/or 3 time periods; for balls attrition time periods was approximately of 2, 6 and 24 hrs., and

for autogenous attrition time period consisted of 24, 72 and 120 hrs. Time periods were set after some trial tests to identify the amount of material broken and to avoid running out of material specially when using steel balls. Material broken (in percentage weight) was identified sieving the sample after the test. Total amount of tests and more detailed data can be seen in Table 3.

Particle shape was measured using eleven shape descriptors or quantities (Table 2) through two dimensional image analysis. Graphical description of the quantities can be found in the appendix. Shape change was statistically identified using Mann-Whitney Test (Moses, 2014) and Two-Sample t-test (Snedecor & Cochran, 1989) with 5% significance level. Since the data distribution is not all the time normal it was decided to use the two mentioned statistical tests, the non-parametric test Mann-Whitney test applies for unknown and skew distributions while two samples t-test determine differences when the data is normally distributed.

Sub-quantities were classified according with the authors (Rodriguez, 2012); form is recognized by the “#” and roundness by “+”symbols in Table 2.

Table 2 Quantities use to determine the particle shape

Quantity	Description	Working range	Reference
1	+4πArea/Perimeter ²	0-1	(Cox, 1927)
2	#4Area/πMajor axis ²	0-1	(ImageJ, 2013)
3	+Area/Convex Area	0-1	(Mora & Kwan, 2000)
4	+Fractal dimension	1-2	(Image Pro Plus v. 7.0, 2011)
5	#Square root of Maximum inscribed/Minimum circumscribed, circle diameters	0-1	(Riley, 1941)
6	#Diameter of a circle same area as particle/Minimum circumscribed circle diameter	0-1	(Wadell, 1935)
7	#Perimeter ² /Area *	0-1/4Pi	(Blott & Pye, 2008)
8	#Perimeter of a circle with same area/Perimeter	0-1	(Wadell, 1935)
9	#Area/Area of the minimum circumscribed circle	0-1	(Tickell, 1938)
10	+Perimeter/Convex perimeter *	0-1	(Janoo, 1998))
11	+πAverage Feret/Perimeter	0-1	(Image Pro Plus v. 7.0, 2011)

*Inverse was used to obtain a working range between 0 and 1

Quantity describing the large scale shape descriptor “form”

+Quantity describing the intermediate scale shape descriptor “roundness”

4 RESULTS

Table 3 is showing the result of each individual test arranged by sample, milling time and percentage of fines produced. Fine production percentage was measured in relation to the initial sample weight and the original sieve size. Figure 2 summarizes the results from the tests. Dashed lines are those tests that contains iron balls as degradation agent while continues lines are samples in autogenous attrition. The use of balls as an attrition material speed up the degradation process of the tailings (size change) and it is evident when comparing the general slope of dashed and continues lines (balls and autogenous grinding).

Relative shape change results are shown in Figure 3, gray colored markers and lines shows when most of the shape descriptors or

quantities values have changed. Figure 3 left (ball attrition) illustrate that there is a general shape change during all steps for sizes 0.5 and 0.25mm (except last step for 0.25mm).

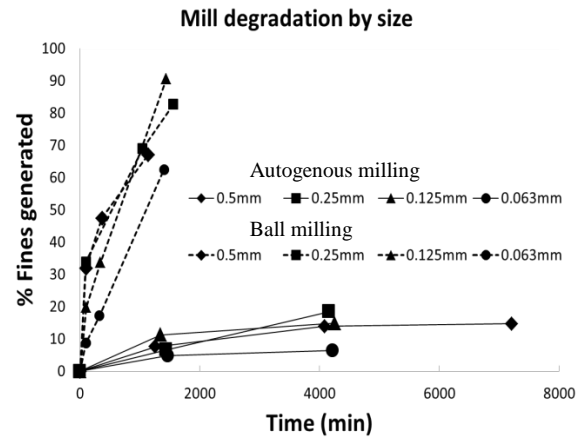


Figure 2 Fines generation in mill test Dashed lines are those tests that contains iron balls as degradation agent while continues lines are samples in autogenous attrition.

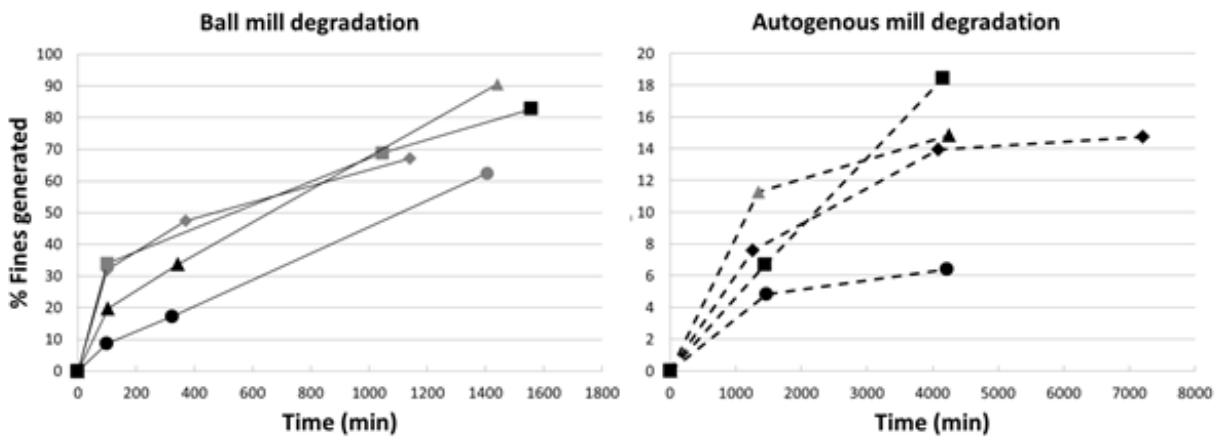


Figure 3 Degradation and shape change by milling agent. Gray leyends indicate shape change. Left ball milling. Right autogenous milling (Fraction sizes are represented by the lower limit)

Table 3 Rolling time periods and percentage of material undergoing the size range (fines generation)

Sample	Time period (min)	% fines generated	Sample	Time period (min)	% fines generated
0.5 _a	1260	7.6	0.5 _b	100	32.0
	4085	13.9		370	47.5
	7207	14.7		1140	67.2
0.25 _a	1442	6.7	0.25 _b	100	33.9
	4158	18.4		1046	69.0
				1558	82.8
0.125 _a	1344	11.3	0.125 _b	103	19.8
	4252	14.8		343	33.7
				1441	90.6
0.063 _a	1470	4.8	0.063 _b	100	8.7
	4222	6.4		324	17.2
				1406	62.5

Subscripts: *a* for autogenous and *b* for balls attrition

Smaller sizes 0.125 and 0.063mm only show shape change in the last step. From Figure 3 right (autogenous attrition) only 0.125mm size has shown shape change.

In same Figure 3 there is a rapid step increase in the broken percentage in all first step time tests. Particles seem to break more rapidly during the first minutes of the tests (see the initial slope in Figure 3 right and left).

Relative shape changes by time step and quantity are shown in Table 4 represented by a gray shadow. When “↓” appears the particle becomes more elongated or angular otherwise they are more regular in form and rounded. Quantities mean values before and after the time step attrition are presented.

Table 4 Relative shape change time-step, data represent the mean value. Shape change is highlighted by a gray shadow when particles become more regular in form or rounded; the appearance of “↓” means the opposite shape change.

S	Q	Balls						Autogenous					
		2hr		6hr		24hr		24hr		72hr		120hr	
A	+1	0.665	0.681	0.681	0.702	0.702	0.732	0.665	0.668	0.668	0.673	0.673	0.672
	#2	0.728	0.742	0.742	0.735	0.735	0.760	0.728	0.743	0.743	0.741	0.741	0.748
	+3	0.924	0.935	0.935	0.943	0.943	0.951	0.924	0.932	0.932	0.932	0.932	0.931
	+4	1.041	1.031	1.031	1.025	1.025	1.022	1.041	1.033	1.033	1.032	1.032	1.031
	#5	0.787	0.802	0.802	0.803	0.803	0.819	0.787	0.796	0.796	0.796	0.796	0.801
	#6	0.786	0.800	0.800	0.799	0.799	0.818	0.786	0.792	0.792	0.792	0.792	0.798
	#7	0.126	0.161	0.161	0.160	0.160	0.160	0.126	0.159	0.159	0.153↓	0.153	0.159
	#8	0.847	0.856	0.856	0.873	0.873	0.887	0.847	0.852	0.852	0.855	0.855	0.851
	#9	0.620	0.642	0.642	0.641	0.641	0.672	0.620	0.631	0.631	0.631	0.631	0.640
	+10	0.929	0.930	0.930	0.946	0.946	0.949	0.929	0.931	0.931	0.933	0.933	0.927↓
	+11	0.296	0.296	0.296	0.301	0.301	0.303	0.296	0.297	0.297	0.297	0.297	0.295↓
B	+1	0.676	0.705	0.705	0.745	0.745	0.746	0.676	0.687	0.687	0.689		
	#2	0.710	0.748	0.748	0.740	0.740	0.771	0.710	0.728	0.728	0.730		
	+3	0.922	0.932	0.932	0.949	0.949	0.948	0.923	0.924	0.924	0.928		
	+4	1.045	1.041	1.041	1.033	1.033	1.033	1.047	1.042	1.042	1.042		
	#5	0.779	0.800	0.800	0.812	0.812	0.822	0.779	0.789	0.789	0.794		
	#6	0.780	0.798	0.798	0.810	0.810	0.823	0.780	0.787	0.787	0.790		
	#7	0.079	0.079	0.079	0.082	0.082	0.082	0.079	0.082	0.082	0.080		
	#8	0.852	0.872	0.872	0.900	0.900	0.902	0.852	0.858	0.858	0.862		
	#9	0.612	0.640	0.640	0.659	0.659	0.680	0.612	0.624	0.624	0.629		
	+10	0.937	0.947	0.947	0.964	0.964	0.963	0.937	0.940	0.940	0.940		
	+11	0.299	0.302	0.302	0.307	0.307	0.307	0.299	0.300	0.300	0.300		
C	+1	0.700	0.703	0.703	0.702	0.702	0.643↓	0.700	0.684↓	0.684	0.696		
	#2	0.706	0.713	0.713	0.710	0.710	0.689	0.706	0.708	0.708	0.719		
	+3	0.936	0.935	0.935	0.939	0.939	0.919↓	0.936	0.926↓	0.926	0.934		
	+4	1.034	1.032	1.032	1.031	1.031	1.038↓	1.034	1.032	1.032	1.032		
	#5	0.784	0.785	0.785	0.787	0.787	0.769↓	0.784	0.778	0.778	0.789		
	#6	0.780	0.784	0.784	0.784	0.784	0.769↓	0.780	0.777	0.777	0.788		
	#7	0.038	0.040	0.040	0.040	0.040	0.039	0.038	0.046	0.046	0.044		
	#8	0.878	0.877	0.877	0.878	0.878	0.843↓	0.878	0.865↓	0.865	0.872		

#9	0.614	0.619	0.619	0.620	0.620	0.597↓	0.614	0.610	0.610	0.625
+10	0.961	0.960	0.960	0.960	0.960	0.936↓	0.961	0.953↓	0.953	0.953
+11	0.307	0.306	0.306	0.306	0.306	0.298↓	0.307	0.304↓	0.304	0.304
D										
+1	0.700	0.702	0.702	0.713	0.713	0.690↓	0.698	0.713	0.713	0.713
#2	0.685	0.684	0.684	0.714	0.714	0.688	0.685	0.692	0.692	0.693
+3	0.931	0.933	0.933	0.937	0.937	0.925↓	0.931	0.939	0.939	0.936
+4	1.051	1.051	1.051	1.053	1.053	1.061↓	1.051	1.051	1.050	1.050
#5	0.772	0.773	0.773	0.786	0.786	0.769↓	0.772	0.777	0.777	0.781
#6	0.768	0.771	0.771	0.782	0.782	0.766↓	0.768	0.772	0.772	0.778
#7	0.024	0.023	0.023	0.022↓	0.022	0.018↓	0.024	0.023	0.023	0.024
#8	0.875	0.876	0.876	0.885	0.885	0.869↓	0.875	0.885	0.885	0.883
#9	0.596	0.600	0.600	0.616	0.616	0.593↓	0.596	0.602	0.602	0.609
+10	0.964	0.965	0.965	0.967	0.967	0.959↓	0.964	0.971	0.971	0.967↓
+11	0.308	0.308	0.308	0.309	0.309	0.306↓	0.307	0.310	0.309	0.308↓

S=Sample size, Q=Quantity number, ↓=less regular in form, less rounded

Table 5 Shape change compared with initial state. Mann-Withney and Two-sample t-tests comparison. Shape change is highlighted by a gray shadow when particles become more regular in form or rounded; the appearance of “↓” means the opposite shape change.

Size	Q	Balls						Autogenous						
		2hr		6hr		24hr		24hr		72hr		120hr		
		MW	Tt	MW	Tt	MW	Tt	MW	Tt	MW	Tt	MW	Tt	
A	+1													
	#2													
	+3													
	+4													
	#5													
	#6													
	#7													
	#8													
	#9													
	+10													
	+11													
B	+1													
	#2													
	+3													
	+4													
	#5													
	#6													
	#7													
	#8													
	#9													
	+10													
	+11													
C	+1						↓		↓					
	#2													
	+3						↓		↓					
	+4						↓							
	#5						↓							
	#6													
	#7													
	#8						↓		↓					
	#9													
	+10						↓		↓				↓	
	+11						↓		↓				↓	
D	+1													
	#2													
	+3						↓							
	+4						↓							
	#5													
	#6													

#7				↓		↓				
#8										
#9										
+10						↓				
+11						↓				

Q=Quantity number; MW=Mann-Withney test, Tt=Two samples t-test

Table 4 shows that tailing particles subject to ball attrition in sizes 0.5 and 0.25 mm in general become more rounded and regular in form as the time-step evolves while for particle sizes 0.125 and 0.063 mm practically no change is seen until last time-step where increase in angularity and irregularity in form is perceived.

For autogenous attrition time-step changes are not recognized in the majority of the quantities (as in ball attrition) for coarse sizes 0.5 and 0.25 mm, however fine fraction 0.125 and 0.063mm present changes in the first time-step with special remark on size 0.125mm that became more angular and irregular in form. Table 5 is showing the shape change with respect to the initial state of the particles. In this table gray cells identify the shape change with respect to the original shape. Mann-Whitney Test (Moses, 2014) and Two-Sample t-Test (Snedecor & Cochran, 1989) are located together to evaluate differences, practically results agree in both tests.

5 DISCUSSION

Mill attrition test were performed for different fraction sizes (Table 1) during different time periods (Table 3). Attrition degradation was identified by sieving (fines generation) and particles shape change by image analysis.

Tailing particles images were also subject to visual inspection and tailings had been classified as very angular to sub angular material base on Powers (1953) comparison chart The results of this classification is in agreement with the conclusion of the general shape of tailings made by e.g. Garga, et al. (1984). Visual inspection of the original material (splitted by size) in the soil containers detects particle breakage; particles are weak enough to be eroded during the handling and sample preparation. This could affect the results since an already broken

portion of material could be introduced in the mill. Lade, et al. (1996) suggest that angular particles are very susceptible to break even at low stresses because they concentrate in the angular contact points. Furthermore the particle breakage is also a function of time even under constant stresses (Yamamuro & Lade, 1993). That could explain why sample 0.25mm in figure 3 (left) shows an outstanding breakage at the end point (around 4000 minutes of the running test). It is also shown that particles are broken at higher rate in the initial step.

During this study two statistical methods were used, Mann-Withney test and two sample t-test. Mann-Withney test is applicable to skew distribution where the normality test is not recognized. Two samples t-test is used when the normality distribution of the data is accomplished. Data in the study was detected to be in some cases normal distributed and a comparison of results among these methodologies is of interest (see Table 5). Even if the distribution of the data is not normal results has shown that there is a 96% of coincidence between the two statistical methods. Furthermore the shape change detection is not skew to one of the methods or in a specific quantity. Two samples t-test could be used to determine differences in shape since there is no evidence that the skew distribution of the data affect the results.

Quantities Table 2 are identified as a form or roundness descriptor as Mitchell and Soga (2005) sub-quantities classification (according to authors). The classification is not showing any relation with the attrition results, shape changes are seems not to be related with the shape sub-quantities classification (Figure 1). Choose of the best quantity or shape descriptor is of interest and attempts can be found in literature (Bouwman, et al., 2004) Even if there are some standards e.g. ASTM D3398, D4751 they are only there to avoid unfavorable

shape (e.g. elongated particles) but still there is no general agreement on which shape descriptor, or descriptors, should be considered universally. In the scientific literature some quantities appear more than others e.g. Wadell (1932) comparison chart (recently computerized by Zheng & Hryciw, 2015), aspect ratio (Hawkins, 1993), circularity (Cox, 1927) among others.

Soil strength is shape dependent as Santamarina & Cho (2004) conclude, in perspective the gain in angularity for the smaller fractions (0.125 and 0.063mm) could increase the soil strength but reduce it in the coarser fractions. Any way it is unknown on which size has the major influence thus, more studies should be necessary.

Ulusoy (2008) study shows that ball milling produces more angular and irregular particles opposite of what this study concluded. Furthermore his quantity values were always lower indicating that produced particles are more angular and irregular in form compared with this research. There are two main differences that could affect the results; the mineralogy and the milling process itself. Talk mineral was used by Ulusoy (2008); talk has the lower value in the Mohr hardness scale. The Ulusoy (2008) milling intention was to break the particles while this study the attrition was the objective; it is known that the final shape of the particles is considered to be the result of several factors among them the rigor of the transport (Wentworth, 1922).

6 CONCLUSIONS

Breakage occurs in all states even in sample deposits (bags, trays)

Particles with ball attrition eventually change their initial shape to be more rounded/circular in sizes 0.5 and 0.25mm but fine particles 0.125 and 0.063mm come back to be angular/irregular.

Attrition is more intensive when erosional agents as iron balls are included.

It is possible to use any of the statistical methods Two samples t-test or Man-Withney since there is practically no difference in the results.

There is no skewed shape change difference in between the sub-quantities morphology and roundness.

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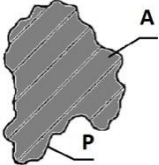
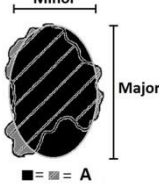
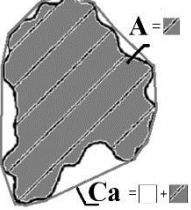
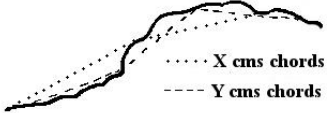
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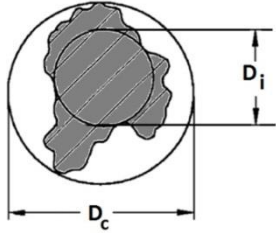
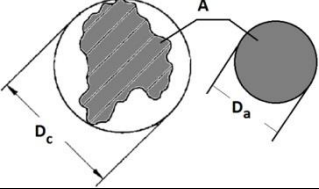
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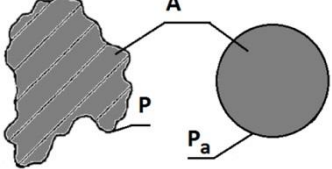
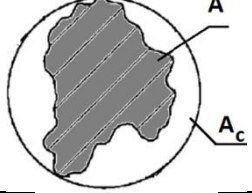
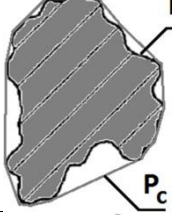
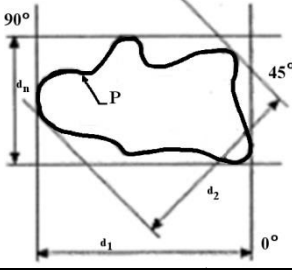
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APENDIX

Q	Description	Graphic description
1*	$4\sqrt{\text{Area}(A)/\text{Perimeter}^2(P)}$	
2*	$4\text{Area}(A)/\sqrt{\text{Major axis}^2(\text{Major})}$	
3*	$\text{Area}(A)/\text{Convex Area}(Ca)$	
4	Fractal dimension Fractal dimension use 'strides' (minimum step lengths) of various sizes. The fractal dimension is calculated as 1 minus the slope of the regression line obtained when plotting the log of the perimeter (for various strides) against the log of the stride length. (more info in imageproplus)	

5*	Square root of aximum inscribed (Di)/Minimum circumscribed(Dc), circle diameters	
6*	Diameter of a circle same area as particle(Da)/Minimum circumscribed circle diameter(Dc)	

7	$\text{Perimeter}^2(P)/\text{Area}(A)$	See figure in quantity 1 in this table
8*	Perimeter of a circle with same area (Pa)/Perimeter(P)	
9*	Area(A)/Area of the minimum circumscribed circle (Ac)	
10*	Perimeter/Convex perimeter	
11	$\pi \text{Average Feret}/\text{Perimeter}(P)$ Average feret box is obtained rotating two parallel lines (two degrees each time) and measuring the distance, finally the average feret is the average distance of all the feret boxes distance measured	

* Figures were taken and modified from Johansson and Vall 2012