Statistical analysis of thaw index for thaw weakening design purposes

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ABSTRACT
In frost action research and design most focus has been put on frost heave. There are several models available to predict frost heave and the concept of statistical re-occurrence times of frost index is commonly used in design. Less effort has been put on thaw weakening modeling and design even though thaw weakening causes a lot of damage on infrastructure such as road and railways. In e.g. Sweden the Transport Administration collects temperature data during the freezing season for frost heave design purposes on project levels but thawing effects are conventionalized by reference tables on a regional level.

The aim of this study is to define and quantify the thawing event in such way that it could be applied on a project level in a similar way as temperature measurements are used for frost design. In this study thaw index has been analysed statistically based on air temperature data from 50 stations, collected from year 1951 and onwards by the Swedish Meteorological and Hydrological Institute (SMHI), in Sweden. The temperature thaw seasons have been correlated to the Swedish Transport Administrations frost depth measurement stations.

A reference table has been developed based on reoccurrence times of thawing seasons based on the magnitude of thawing index. Based on the statistical distribution of thawing indexes Sweden was divided into three regions in this study. It seems, by statistical means, to be fully possible to develop reference values for thawing periods where thawing index is the working unit. In order to implement the results in e.g. pavement design thawing temperature measurements on a local or semi-regional level are required.

Keywords: Thaw weakening, pavements, temperature analysis

1 INTRODUCTION

In seasonal frost regions frost action has major effect on pavement lifetime. Frost heave causes cracking of the pavement surface and unevenness on the road surface and thaw weakening decreases the bearing capacity. Both of these phenomena decrease the pavement life time, Berglund (2009).
In frost action design often the concept of an estimation of the frost heave based on a design winter temperature, commonly expressed as frost index (*FI*), from a nearby located metrological station. Thaw weakening design is in general more based on best practice. In e.g. the Swedish framework the thaw weakening is encountered for by seasonal adjustments of the stiffness modulus in the pavement design procedures on a regional level, Trafikverket, (2011).

Thaw weakening on roads is effected by a range of varaibles, e.g. temperature at freezing and thawing, soil type, the access to water in the freezing and hawing season, the local draining conditions and the traffic, Phukan, (1985). The variation of the degree of thaw weakening is in high degree related to the temperature gradient in spring time. High temperature gradients results in severe thaw weakening and at low temperature gradients the thaw weakening is less severe, Berglund (2009).

Design charts for frost depth and design frost index has been used in pavement and foundation design for decades, e.g. Stål and Vedel, (1984). Similar charts or analysis has so far not been adapted for thaw weakening. In this study temperature from Sweden has been analysed statistically during thaw events for over 50 years in order to propose design criteria for thawing periods. Variation and re-occurrence of temperature scenarios has been studied. A concept of forecasting thaw weakening based on temperature readings and historical temperature data is proposed based on a model suggested by Hicks et al. (1985). Thaw weakening may be divided into surface weakening, affecting the uppermost layers in the superstructure, and deap weakening, affecting the subgrade, Gandahl, (1987). Hicks et al. (1985) and this study focuses on the deap weakening in the subgrade only.

2 METHODOLOGY

2.1 Outline of study

In this study long time series of air temperature data has been used for statistical analyses of the variation thawing events in spring time and its spatial distribution in Sweden. To model the thawing event an air temperature based model proposed by Hicks et al. (1985) has been used. The results have been validated by monitoring data from frost depth measurements. Since the analysis not is site specific a range of simplifications has been used; the thaw weakening is only regarded in the subgrade, thermal properties are considered to be constant etc.

2.2 Air temperature data

The air temperatures used in this study are open climate data provided by the Swedish Metrological and Hydrological Institute (SMHI, 2015). Data from 50 temperature stations in Sweden from 1951-2014 has been analysed. The locations range from Malmö in the south to Luleå in the north of Sweden. The location of the stations is shown to the left in figure 1.

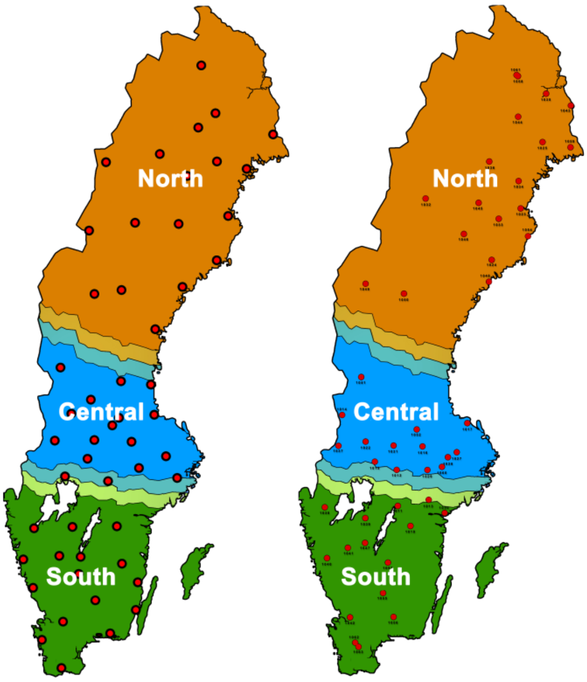

Figure 1 To the left: Location of the SMHI air temperature monitoring stations used in this study. To the right: Location of the Swedish Transport Administrations thaw depth monitoring stations used in this study.

2.3 Frost depth monitoring data

Monitoring data from the Swedish Transport Administrations thaw depth measurement stations has been used to study spatial variations in thaw weakening over the country and to validate the results from the statistical analysis of the air temperatures. There is totally 54 thaw depth stations, of 48 are in operating status, monitoring thaw depth since 2006, Trafikverket, (2015). The operating stations are displayed on the map to the right in figure 1. The stations monitors thaw depth 12-24 times a day at an intermediate distance of 5 cm between the sensors, down to 2 m depth below the surface. In this study monitoring data has been aggregated to daily average values. The monitoring stations of frost depth lack air temperature monitoring. In order to correlate the frost depth readings by air temperatures has air temperature data from adjacent weather stations for road maintenance been use (VViS-stations).

2.4 Analysis of frost depth and thawing seasons

The monitoring data from the frost depth analysis has been used to quantify the length of the thawing season. The length of the thawing season is defined between the starting date when the upper temperature reading > 0˚C until the whole soil profile is > 0˚C. The thawing period for each station is compiled by an average time length *t* based on the all the recorded seasons.

The average length of the thawing season of each station has been correlated to the geographical position (here in SWEREF) by linear regression in the program MINITAB 17, Minitab, (2015) in order to explore the variation in thawing season from the south to the north of Sweden. Based on the results the monitoring stations are grouped into regions for further analysis. The average regional thawing period is calculated and further on referred to as t_{max}. The time is used for computation of TI_{acc} for each region, i.e. the thawing index needed to thaw the subgrade in the region.

2.5 Thaw model

Hicks et al. (1985) proposes a thaw model based on thawing index. The thawing index *TI* is the accumulated air temperature above the freezing relative a reference thawing temperature. The procedure for calculating the thawing index *TI* follows the procedure presented in Berglund et al. (2011).

$$\mathrm{TI} = \left(T_m - T_{ref}\right) \cdot \Delta \mathrm{t} \qquad (1)$$

Where; T_m is the average daily temperature, T_{ref} is the temperature at thawing starts (here 0 ˚C), and Δt time step (here 1 d) in the thawing period. The thawing index is accumulated over time in the thawing period, TI_{acc}. To account for intermediate freezing events the accumulated thawing index decreases by half of the freezing index *FI* (Hicks et al. 1985), equation 2.

$$TI_{acc} = \sum(TI - 0.5 \cdot FI) \qquad (2)$$

When $T_m \leq 0$˚C the freezing index *FI* is calculated according to equation 3.

$$\mathrm{FI} = \sum(0 - T_m) \qquad (3)$$

The start of the thaw period id defined when a limit value of the accumulated thaw index ($TI_{acc,lim}$) has been exceeded. In this study has the proposed thawing index limit by Hicks et al. (1985) been used. For thicker pavements, comparable to engineered roads, the proposed limit is $TI_{acc,lim} \approx 14$ ˚d (approximation from Fahrenheit). An additional condition for defining the start of a thaw period in this study to have subsequent positive daily average air temperatures after $TI_{acc,lim}$ has been reached.

To determine if frost action may be present Stefan's formula has been used. Based on a thickness of 0.5 m superstructure at least a frost index $FI \geq 50$˚d is needed.

2.6 Data analysis

Thaw index is calculated for all the SMHI climate stations there complete data series are present. The starting date for the individual season is defined as when $TI_{acc,\ lim}$ for the station. T_{acc} are the calculated up to t_{max}, the defined end of the thawing season. For stations where several thawing seasons has been observed in the frost depth measurements more than one thawing season are modelled if the temperature data implies so.

Statistical data analysis has been applied in the progress of the thawing index. The analysis has been performed on the individual analysis of thaw index from *t*=3 days to t_{max}. The choice to start the analyses at day t=3 are based on to have a continuous thawing event in progress and to avoid effects of extreme temperatures during a very small time interval.

In the statistical analysis the computed T_{acc} for each station, season and day *t*=3 to t_{max} are used as data series [x_1: x_n]. A continuous statistical distribution fitting the data set is needed in order to be able to compute reoccurrence, Benjamin & Cornell, (1970). The reoccurring thaw index TI_{acc} is denoted x_t where the index t denotes the time unit. In this study the time unit is number of thawing seasons per year. If the dataset belongs to the continuous statistical distribution *F(x)* (Castillio, 1988):

$$\mathrm{F(x)} = \mathrm{P}(\mathrm{X} \le x_t) = \frac{1}{t} = p \qquad (4)$$

The statistical distribution gives the probability of the stochastic variable X to have a value less than x_t. In this case is the complementary outcome of interest.

$$P(X \ge x_t) = 1 - F(x) = F_c(X) \qquad (5)$$

or

$$\mathrm{t} = \frac{1}{F_{c(X)}} \qquad (6)$$

Based on the statistical distribution *F(X)*, or in this case the complementary distribution function, x_t, can be computed from equation 6.
The distribution *F(X)* has been analysed, based on the TI_{acc}-values by the software EasyFit, Mathwave, (2015). Three possible distributions were identified as possible: Burr type XII, Johnson S_b, Johnson S_u, All of these distributions requires four input parameters, Johnson & Kotz, (1970). To determine the best fit of distribution the Anderson-Darling test was used.
Based on the chosen distribution the re-occurrence of x_t for $t \in [2, 5, 10, 15, 20, 25, 30, 50, 75, 100, 150]$.

2.7 *Evaluation of thaw events*

In this study the thaw events have been classified the number of thawing periods per season.

3 RESULTS

3.1 *Thaw periods*

The average thawing period (t_{max}) for all individual stations are plotted vs. the latitude coordinate in figure 2. The inserted trend line shows a high correlation between the length of the thawing season and the latitude, R^2=0.82. As seen in the figure the variation increases in the north for the length of the thawing period.

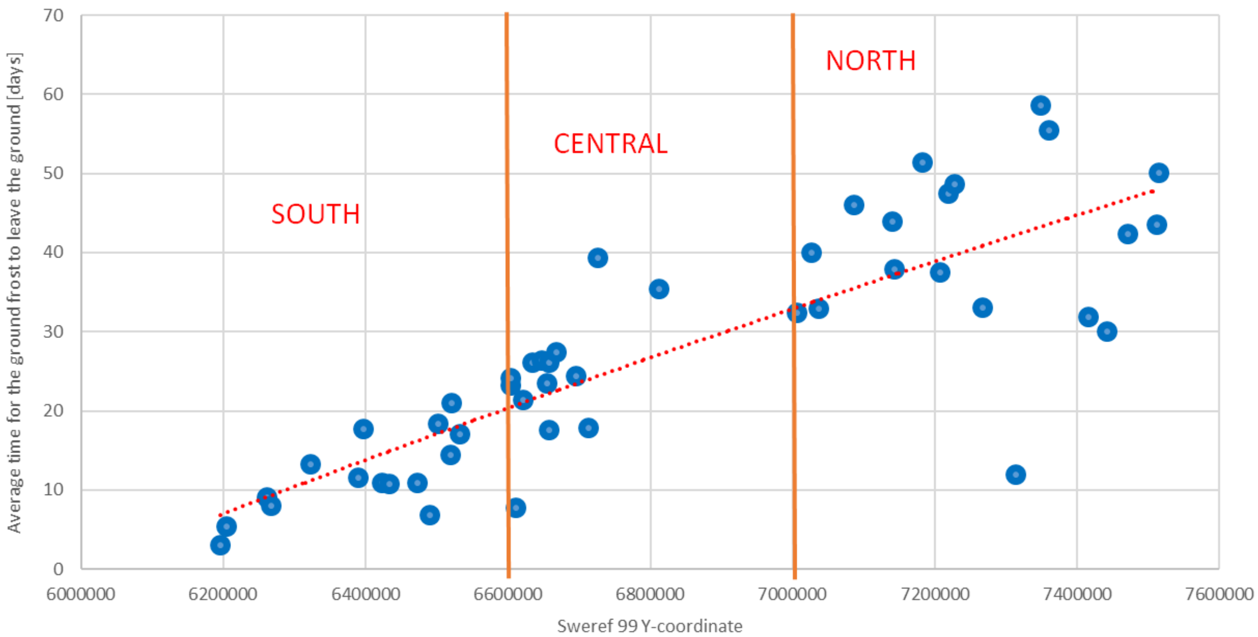

Figure 2 Average time t_{max} in days for the thawing period vs. the Y-coordinate in the SWEREF-coordinate system (from south to north).

Based on the results in figure 2 the stations were divided into three different regions; north, central and south, indicated in figure 1 and 2. In table 1 is the average thawing periods t_{max} presented for the three regions.

Table 1 The average thawing periods t_{max} for the three different regions.

Region	t_{max} [d]
North	41
Central	24
South	12

The number of years included in the analysis and the average number of thawing periods for the three regions are summarised in table 2.

Table 2 The average thawing periods t_{max} for the three different regions.

Region	No. years of data	No. of thawing periods
North	822	1.23
Central	831	1.11
South	831	1.04

EVALUATION OF THAWING

Reference values ∑(TI-0.5FI)

Thawing index evaluation

Region (as per map)

North

1.05 Seasons

$1-F(X)^{-1}$ [år] \ Δt [d]	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
T2	9,6	12,1	14,6	17,1	19,6	22,1	24,6	27,2	29,8	32,7	35,9	39,2	42,7	46,8	50,1	54,2	58,6	64,5	68,5	73,6	78,8	84,2	89,8
T5	14,6	19	23,4	27,6	32,1	36,6	40,9	45,2	49,7	54,4	59,5	64,9	70,6	75,9	82,4	88,8	95,6	103,8	110,2	117,8	125,4	133,3	141,2
T10	17,3	22,8	28,3	33,7	39,2	45	50,6	56	61,5	67,3	73,4	79,9	86,7	92,5	100,7	108,2	115,9	125,1	132,6	141,2	150	159	168
T15	18,6	24,7	30,9	36,8	43	49,5	55,7	61,7	67,8	74,1	80,8	87,9	95,1	101,4	110,1	118,1	126,4	135,9	143,9	152,9	162,1	171,6	181,2
T20	19,5	25,9	32,5	38,8	45,5	52,5	59,2	65,7	72,1	78,8	85,8	93,3	100,8	107,5	116,4	124,8	133,3	143,1	151,3	160,5	170	179,8	189,8
T25	20,1	26,8	33,7	40,4	47,3	54,8	61,8	68,6	75,4	82,3	89,6	97,4	105	112,1	121,1	129,7	138,4	148,3	156,7	166,1	175,8	185,8	196
T30	20,6	27,5	34,7	41,6	48,8	56,6	64	71	78	85,2	92,7	100,7	108,4	115,9	124,9	133,6	142,4	152,5	161	170,5	180,3	190,4	200,9
T50	21,9	29,4	37,2	44,8	52,8	61,5	69,7	77,5	85,2	92,9	101	109,6	117,7	126,2	135	144,1	153,2	163,5	172,3	182	192,1	202,6	213,5
T75	22,9	30,8	39,1	47,8	55,8	65,3	74,2	82,6	90,7	98,9	107	116,4	124,7	134,3	142,6	151,9	161,3	171,6	180,6	190,4	200,7	211,4	222,7
T100	23,5	31,8	40,3	48,9	57,8	67,9	77,3	86,1	94,6	103,1	112	121,1	129,5	140	147,8	157,3	166,7	177,2	186,2	196	206,4	217,2	228,7
T150	24,3	33	42	51,1	60,6	71,5	81,6	91	100	108,9	118	127,7	136,2	148,1	154,9	164,6	174,2	184,6	193,7	203,5	214	225	236,8

Central

1.11 Seasons

$1-F(X)^{-1}$ [s.] \ Δt [d]	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	Degree of s
T2	9,5	12,1	14,7	17,2	19,6	22,1	24,6	27,1	29,7	32,4	35,2	38,1	41	44	47,1	50,5	54,1	57,9	61,9	66,1	70,4	74,9	1
T5	14,3	18,4	22,7	27,2	31,5	35,9	40,1	44,2	48,5	52,8	57	61,6	66,4	71,4	76,4	81,8	87,4	93,4	99,8	106,5	113,4	120,6	2
T10	17	22	27,2	32,8	38,2	43,7	49	54,2	59,6	65,1	70,4	76	82	88,2	94,4	100,1	107,8	115	122,8	130,8	139,1	147,7	3
T15	18,4	23,9	29,6	35,7	41,6	47,7	53,6	59,5	65,7	71,8	77,7	84	90,7	97,6	104,4	111,6	119,1	126,9	135,3	144	153,1	162,4	4
T20	19,4	25,2	31,2	37,6	43,9	50,4	56,8	63,1	69,8	76,4	82,8	89,6	96,8	104,2	111,4	119,1	127	135,2	144	153,2	162,7	172,4	5
T25	20,1	26,1	32,4	39	45,6	52,5	59,1	65,9	73	80	86,8	93,9	101,5	109,2	116,8	124,8	133,1	141,5	150,7	160,2	170	180,1	6
T30	20,6	26,9	33,4	40,2	47	54,1	61	68,1	75,5	82,9	90	97,4	105,3	113,3	121,2	129,5	138	146,7	156,1	165,8	176	186,3	7
T50	22,1	29	36,1	43,2	50,6	58,4	66,2	74,2	82,6	91	98,9	107,2	116	124,9	133,6	142,5	151,8	161,1	171,2	181,5	192,4	203,5	8
T75	23,2	30,6	38,2	45,5	53,3	61,7	70,1	78,9	88,2	97,3	106	115	124,5	134,1	143,4	153	162,7	172,5	183,1	193,8	205,4	216,9	9
T100	23,9	31,6	39,7	47	55,2	64	72,9	82,2	92,1	101,3	111	120,6	130,6	140,7	150,5	160,4	170,6	180,7	191,5	202,6	214,5	226,4	10
T150	24,9	33,2	41,7	49	57,8	67,1	76,7	86,9	97,6	108,2	118	128,6	139,3	150,1	160,5	171	181,7	192,2	203,4	214,9	227,4	239	11

South

1.23 Seasons

$1-F(X)^{-1}$ [s.] \ Δt [d]	3	4	5	6	7	8	9	10	11	12	Degree of severity
T2	9,5	12,1	14,7	17,4	20	22,5	24,8	27	29,4	31,9	1
T5	14,4	18,4	22,5	26,8	31,1	35,4	39,5	43,5	47,6	51,8	2
T10	17,3	22,2	27,1	32,3	37,7	43	48,3	53,4	58,5	63,6	3
T15	18,8	24,3	29,6	35,3	41,2	47,2	53	58,8	64,4	70	4
T20	19,9	25,7	31,4	37,4	43,5	50	56,3	62,5	68,4	74,3	5
T25	20,7	26,8	32,7	38,9	45,4	52,1	58,8	65,3	71,5	77,6	6
T30	21,3	27,7	33,7	40,2	46,8	53,8	60,8	67,5	74	80,2	7
T50	23,1	30,1	36,6	43,6	50,8	58,6	66,3	73,7	80,8	87,4	8
T75	24,4	32	38,9	46,3	53,9	62,3	70,6	78,6	86	93,1	9
T100	25,4	33,4	40,5	48,1	56,1	64,8	73,6	82	89,8	97	10
T150	26,7	35,3	42,8	50,8	59,2	68,5	77,8	86,7	95	102,5	11

Figure 3 Excerpt from the reoccurrence table. The table contains the TI_{acc}-values at the past number of days in the thawing periods for different reoccurrence periods. The table contains values up to t_{max}=41 for the north region.

3.2 *Statistical distribution*

In general the Johnson S_b-distribution had the best fit on the data with a few exceptions. The station in Kalmar in region south followed a Johnson S_u-distribution instead.

4 DISCUSSION

4.1 *Regional effects on thawing*

As expected the thawing season is longer in the north compared to the south. But the intensity in the thawing is similar independent on the latitude location. The low variation support the simplification used in this study of dividing a large are into a few regional areas as also done in e.g. the Swedish Transportation Administrations frost design guidelines. In this study the effect of location in longitude position has not been investigated since the it in the south part of Sweden is a very high correlation between the latitude position and the variation in the north is hard to explain by statistical means due to lack of monitoring stations. It is reasonable to assume that in the north part of Sweden it would be relevant to divide the thaw weakening into at least two groups

based on the longitude locations (coast land and in-land).
It is interesting that the TI_{acc} is in the similar range for all stations during t<12 d.

4.2 The thaw-index model

The thaw-index model, based on Hicks et al. (1985) is based on several simplifications. In this study these simplification has been implemented fully since the aim has been to study the regional effects on thawing events. To be able to fully use this methodology the value of T_{ref} will need to be adjusted for both the pavement construction thickness and material thermal properties and the subgrade properties. This step needs calibration studies to verify. The error induced in this assumption will be systematic but not affect the overall conclusions.

4.3 Thaw weakening reference table

The reference table provides a guidance if the current season or location is representative for the given latitude position. By monitoring local temperature data the point when TI_{acc} has been reached it can be judged if the thaw weakening is moderate, normal or severe. If TI_{acc} normally is reached early, a consequence of high air temperatures, the site based on is local climate conditions are more subjected to severe thaw weakening than expected at its location.
The first 12 days in the reference table are similar in between the three different regions. After 12 days the variation increases between region central and north. It is possible to merge region south and central’s tables to one.

4.4 Statistical distribution of thawing events

In this study the statistical distribution of thawing events has been studied. In Sweden most of the data follows the normal distribution transformation Johnson distributions. Thus it is easy to statistically handle thawing events if a design tool incorporating thawing will be developed.

4.5 Comparison to the Swedish guidelines for pavement frost design

In the Swedish guidelines, Trafikverket, (2011), regarding frost design of pavements thaw weakening is incorporated by assigning reduced stiffness modulus in the superstructure layers and the subgrade during a time period defined by the location in climate zone 1-5 of the road according to the map in figure 4.

Figure 4 The five climate zones in Sweden used for frost design of pavements. Trafikverket, (2011).

In table 3 the length of the deep thawing season and a rough mapping between the climate zones in figure 4 and the regions in figure 1 are showed. The current guidelines assign longer periods of thawing compared to the results in this study. The longer periods for the thaw weakening season in the guidelines is expected since it also includes the recovery of the stiffness modulus after the thawing has stopped.

Table 3 Comparison between the length of the deep thawing season in Trafikverket, (2011) and the average thawing season found in this study. The table also roughly maps the climate zones in Trafikverket, (2011) to the regions in this study.

Current guidelines					
Climate zone	1	2	3	4	5
t_{max} [d]	15	31	45	61	91
This study					
Region	South		Central	North	
tmax [d]	12		34	41	

If the deep thaw weakening should be included in the design based on the local climate conditions there is a need to separate the deep thawing season and the recovery time since the length of the thawing time is strongly correlated to the latitude, as shown in this study, and the recovery time probably are more effected of the material properties, e.g. Phukan, (1985), Andersland and Anderson, (1978) among others.

5 CONCLUSION

Based on historical air temperature data and frost depth measurements the following conclusions can be drawn in this study about thawing in Swedish roads:

- There is a strong correlation in latitude position and the thawing period length.
- The intensity in thawing is similar independent on the latitude positions
- Thawing follows normal statistical distribution transformations

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