

# Analysis of inclined piles in settling soil

F. Resare

ELU Konsult, Sweden, fredrik.resare@elu.se

A.B. Lundberg, G. Axelsson *ELU Konsult, Sweden* 

# ABSTRACT

The use of raked, or batter, piles is an efficient way to handle horizontal forces in constructions. However, if the soil around the pile settles the structural capacity of each pile is reduced because of induced bending moments in the pile. There is currently no validated method in Sweden to analyse horizontal loading from settling soil. In the current paper a non-linear 3D finite element model is validated against a field test from the scientific literature, and the results are compared to three different beam-spring models. These models consist of a state-of-practice model where a subsoil reaction formulation is used, a model where the soil is considered as a distributed load, and a model with a wedge type of failure. Furthermore, a parametric study is conducted for drained soil conditions where the weight and friction angle of the material are varied. The standard soil reaction model yields an induced bending moment almost three times larger than the one obtained from the field test and the two other calculation methods. The latter beam-spring models should therefore be considered in practical design.

### Keywords: Raked piles, batter piles, inclined piles, settling soil, soil structure interaction.

## 1 INTRODUCTION

Simplified beam-spring models are frequently used in design for simulation of axially and laterally loaded piles, (Erbrich et al, 2010). Such models should contain the main mechanisms controlling the soilstructural system, and the validity of the models should be carried out with experimental or numerical methods. Experimental methods include full-scale and field models, which naturally contains empirical evidence of the validity of the model. The latter is the standard method of verification, and should accompanied with a suitable simplified analytical model leads to a robust design methodology, which could however be relatively conservative.

The use of such simplified models outside normal experience, poses the questions about how well the real field case is modelled. Model simplifications comprise material, geometry and boundary conditions. Any of the factors may not be realistically models, which could lead an overtly safe of unsafe design. Some mechanical mechanisms (e.g. changes in pore pressure and creep) occur over longer timescales, which prohibits field verification of the model before the construction is finished. This is a frequent occurring problem in geotechnical design.

This paper discusses numerical simulation of a simplified beam-spring model for a laterally loaded included pile in settling soil for drained soil conditions, which is assumed to be suitable for the long-term conditions of the soil. The structural design, simplified model ground conditions, numerical model and proposed new models of the particular case are discussed.

## 2 PILE DESIGN CONSIDERATIONS

The Northern Scandinavian ground conditions are characterized by soft soils such as clay and peat, placed on a layer of till deposited on hard rock, (Johannessen & Bjerrum, 1969). A commonly used method to transfer loads for overlying structures to the hard rock is to use relatively slender piles that are installed by driving the pile base into the till layer or drilling the pile base into the rock. These end-bearing piles are in most cases dynamically tested to control the geotechnical bearing capacity, i.e. the capacity of the end-bearing rock or till beneath the pile.

This particular piling method results in very high utilization of the structural capacity of the pile. In many cases this factor limits the allowable load on the pile. Design calculations for these piles are normally carried out by calculating the buckling load and structural strength, following the procedure described by Bernander & Svensk, 1970. This calculation method has proven to the robust and has resulted in a safe design.

Such end-bearing piles carry a relatively high load: in the case of a concrete-filled 140 mm steel tubular pile with a wall thickness of 8 mm, the allowable load can exceed 1.2 MN, which makes the foundation system very efficient by using a small amount of steel and concrete. But the small diameter of these piles results in a limited lateral pile bearing capacity; especially since the top layer of the soil frequently consist of very soft clay. Inclined piles are instead preferred as a structural solution, where an inclination of 4:1 to the vertical axis is typically used as a maximum inclination. Pile groups subjected to high levels of horizontal load, e.g. a bridge abutment, therefore normally include a large number of inclined piles.

# 2.1 Alternatives for modelling and design of realistic pile-soil mechanism

Current design methods for the structural strength of piles in settling soil consist of a simplified beam-spring model, outlined in Svan & Alén, 2006 (and Reese et al, 1974, for vertical piles), or a full 3D-FEM-model. The beam-spring model results in limited calculation time and is frequently used in design. 3D-FEM models have not been of extended practical use for the current types of slender piles, although frequently used for lateral loading of larger offshore piles, e.g. Erbrich et al, 2010.

The simplified beam-spring models are obviously a simplification of the soil conditions around the pile, and result from an idealisation in 2D of the soil. The real soilpile deformation mechanism in 3D can be assessed in laboratory, field and numerical

experiments. Because of the relatively complicated processes governing the soil behaviour, including soil settlement, consolidation and creep, field experiments are the most reliable method of assessment. In the scientific literature, field experiments have only been covered in Takahasi (1985). Laboratory models are also discussed in Takahasi (1985) along with both numerical and analytical calculation models. Laboratory experiments are also discussed in Kohno et al, 2010 and Rao et al, 1994. Numerical studies include boundary element models in Poulos, 2006, where the significance of the horizontal load on the piles are discussed in detail, but where a relatively simplified model in adapted to the pile-soil interaction.

The field, laboratory and numerical models mentioned above all detail the resulting bending moments for inclined piles, and a beam-spring model for inclined piles is proposed in Takahasi et al, 1985 and Kohno et al, 2010. No comparison between a full range of soil parameter (friction angle and effective weight) has however been carried out, since this required a large number of simulations, which was outside the scope of these scientific works. In the current paper a numerical model is therefore adapted to an inclined pile, and different beam-spring models are compared to the simulations.

### 3 CURRENT BEAM-SPRING CALCULATION MODEL

The calculation model used in Sweden today (Svahn and Alén, 2006) is based on the equation for a beam on an elastic foundation. The force distribution in the beam on an elastic foundation can be described as a fourth-order differential equation, Equation 1, by dividing the beam in infinitely small elements.

$$EI y^{IV} + N y^{II} + D k_{v} y = D k_{v} y_{a} - N y_{i}^{II} (1)$$

Where EI is the bending stiffness of the pile, N is the normal force along the pile axis, D is the width of the pile,  $k_y$  is the soil reaction transversal to the pile,  $y_g$  is the ground settlement, and  $y^i$  is the i-th differential of the horizontal position along the pile axis x. Equation 1 can be simplified to Equation 2 given that the normal force can be assumed to be constant along the pile, which a suitable assumption for an end-bearing piles. The interesting part of the pile is situated at the pile head where the effective stress is relatively low, which makes this assumption relatively correct.

$$EI y^{IV} = D k_y (y_g - y)$$
(2)

In the current calculation model the position of the soil is a function of the depth to replicate the displacement of the soil. The relative movement of the soil is illustrated in Figure 1.



Figure 1 Illustration of the difference in displacement between the settling soil and the pile causing lateral earth pressure.

# 3.1 Description of the Swedish standard

In the current calculation model the subgrade reaction,  $k_y$ , is set according to empirical values recommended by Reese et al. (1974). The settlement is adapted as a relative movement between the soil and the pile. The settlement is divided into a transversal and a longitudinal component, see Figure 2. From this the transversal part is applied as the movement of the soil and the differential equation can be solved.



Figure 2 Illustration of the division of the settlement into a transversal and a longitudinal component.

This beam-spring model has been used to calculate the bending moment resulting from the settlement and soil conditions in the field experiments in Takahashi, 1985. The field experiment consisted of soil settlement resulting from deposition of fill on soft clay made for a road structure. Measurements were carried out on instrumented pipe piles driven in inclined pairs along the road. The calculated bending moment, according to Svahn and Alén, 2006, along the pile compared to the measured bending moment can be seen in Figure 3. This Figure also shows that the bending moment is larger close to the top of the pile. In the rest of this article only the maximum value of the bending moment will be referred to, however the distribution of the bending moment is relatively similar for all cases, i.e. at the top part of the pile. It should also be noted that the different between the measured and calculated values of the bending moments seems to be related through a scale factor, signifying the extra moment resulting from the idealization of the 3D pile-soil interaction to a 2D beam-spring model.

There are also some limitations present in the 2D analytical beam-spring model, e.g. only one type of soil can be used for the entire length of the pile. Another limitation is that the settlement profile is fixed and does not always represent the actual settlement, since this has to be simplified to an analytical function, e.g. an exponential function.



Figure 3 Comparison between the bending moment as calculated with the current method (Svahn and Alén, 2006) and the bending moment as measured in the full scale experiment by Takahasi (1985).

### 4 ALTERNATIVE BEAM-SPRING MODELS

The comparison between the measured and calculated bending moments in Figure 3 displays that the current model (Svahn and Alén, 2006) clearly overestimates the measured bending moment. This beam-spring model is therefore compared to two different calculation models, described below. The common principle of the two models is that the pressure against the pile is reduced, which means that the force does not exceed the weight of the soil above it. This is a suitable principle to avoid stress distribution resulting which no natural base that result from the simplification of the model.

## 4.1 Distributed load approach

The distributed load approach is originally discussed in Takahashi, 1985. The model results in a division between the top part of the pile and the following lower part along the principle in Randolph, 2014, to represent the real soil response resulting for the different boundary conditions. The soil is therefore divided into two parts; a distributed load part, and a subgrade reaction. The load is applied as a function of the pile width and is limited to the subgrade reaction of the soil so that no load will be applied if the displacement of the pile is equal or limited to the soil displacement, which is more suitable than a load resulting only from the friction angle and effective stress in the traditional earth pressure approach in the current calculation model. The formulation of this behaviour is summarized in Equation (3). This discretization assumes that the soil goes to failure and therefor becomes a load hanging on the top part of the pile.

$$EI y^{IV} = D k_y (y_g - y) \le 3 D \gamma z \sin(\theta) \quad (3)$$

where  $\gamma$  is the weight of the soil and  $\theta$  is the inclination of the pile relative to the vertical axis.

### 4.2 Wedge failure approach

Based on a failure mode described by Reese et al. (1974) the top part of the soil is considered to have a cone like plastic failure which is also observed in the 3D finite element model, discussed below. Similar to the distributed load approach the soil is divided into a distributed load along the top of the pile, and a subsequent subsoil reaction along the deeper parts of the pile. The load is suggested to grow with the weight of the cone shown in Figure 4. Furthermore it is assumed that only the transversal part of the weight will act lateral to the pile causing bending moment (and not the load resulting from increased shaft friction), thus only this part of the weight is to be taken into account. The total load acting on the pile can therefore be described as Equation 4.

 $EI y^{IV} = D k_y (y_g - y) \le$  $\sin(\theta) (\gamma \alpha z^2 \tan(\beta) + \gamma z \tan(\beta) D)$ (4)



Figure 4 Illustration of the wedge failure and parameters used for Equation 4.

# 5 3D FEM MODEL

In order to assess the real behaviour of the soil, either laboratory models (Kohno et al, 2010), numerical models (Poulos, 2006), or field models (Takahashi, 1985) are possible approaches. In the current scientific work a 3D finite element model was used to validate the different 2D-models. The advantage of a numerical model is that parameter studies are possible, and the behaviour of the real case can be studies for different configuration without the limitations of a laboratory or field model, (Randolph, 2014). The current analysis was performed in a 3D FEM software (Hibbitt, Karlsson, & Sorensen. (2001)). The computer model was first validated against the field study in Takahashi, 1985. Subsequently a parametric study was carried out to compare the behaviour of the 3D-FEM model to the different and 2D beam-spring formulations.

# 5.1 Geometry, boundary conditions and FE discretization

Figure 5 represents the geometry of the model, consisting of the pile and the surrounding soil. A symmetry plane along the pile axis has been used to save computational time. The boundary conditions were set so that no displacement perpendicular to the surface will occur except for the top surface which was free to move, and the plane of symmetry where symmetrical conditions were applied.



Figure 5 Basic geometry used for the finite element model.

The soil was modelled as 3D solid elements (C3D8 and C3D4 type of elements, (Hibbitt, Karlsson, & Sorensen. (2001)) and the pile was modelled as shell elements (S4 type of elements, Hibbitt, Karlsson, & Sorensen. (2001)) in order to save computational time during the simulation. As settlement per definition is pore water dissipation, drained parameters were used for the clay.

To simulate the settlement a stress-free strain level was induced in the soil body causing the desired settlement profile. The settlement in the soil was modelled using orthotropic temperature dependency hence shrinking the soil in the vertical direction to represent the settlement profile from the experiment. This resulted in a controlled deformation, in which the load against the pile was controlled by the effective stress level in the soil. The excess pore pressure was consequently not included in the model, but since most of the bending moments occur close to the pile head (according to Figure 3), this should have a relatively small influence on the calculation results, possibly resulting in an overestimation of the bending moments in pile compared to the short-term process, in which less settlements occur and the beginning of consolidation. The pile's top was restrained to move in the horizontal direction to represent the pinned condition from the study. Interaction between the pile and the surrounding soil was modelled using penalty type interface. For the normal behaviour a small pretension was applied between the soil and the pile by changing the clearance when contact pressure is zero and for the tangential behaviour a friction coefficient of 0.385 was assumed (Helwany, 2007). This is a suitable estimate following standard values of the interface friction angle, (Randolph, 2014). The behaviour of the steel is assumed to be linear elastic and a Young's modulus of 200 GPa was assumed along with a Poisson's ratio of 0.3.

## 5.2 Validation of field measurements

The numerical model was calibrated against the field measurements presented in Takahashi, 1985. The field measurements consisted on settlement in a clay soil covered by fill material. The soil was modelled according to the guidelines presented in Trafikverket (2011a), which are normally used in practical design. The elastic properties of the soil were assumed to be linear and isotropic with a Young's modulus calculated as 250 cu for the clay and 50 MPa for the fill material in the embankment. The plastic behaviour was modelled using Mohr-Coulomb plasticity with a friction angle of 45 degrees for the embankment. As settlement in this case per definition occurs due to the dissipation of water drained parameters were used for the clay body, and an alternating value for the clay to estimate the impact of this value (since no drained parameters were presented in Takahashi, 1985). The friction angle of the clay however had very low effect on the results. Furthermore a Poisson's ratio of 0.3 was assumed for the soil body.

### 5.3 Parametric study

In order to compare the different 2D beamspring formulations, (Svahn and Alén, 2006), Reese et al, 1974 and Takahashi, 1985), a parametric study comparing the 3D FEM numerical model to different 2D beam-spring model was conducted. The settlement profile in the 3D-model was predefined, so that the settlement increased linearly over the entire depth. A similar settlement profile was used in the beam-spring model. The density of the soil was set to 0.5, 1.2, and  $2 t/m^3$  and the friction angle was set to 25, 35, and 45 degrees for a total of 9 combinations, shown in Table 1. The Young's modulus of the soil was set to 50 MPa for the entire depth and a cohesion of 2 kPa was used to prevent numerical problems close to the surface. This results in a simplification of the soil parameters, since the modulus tends to increase with the friction angle, but this was not considered in the model. A total settlement of 25 cm was induced and the maximum bending moments were calculated in the 3D-FEM model and the beam spring models, following the distribution of bending moment shown in Figure 3. The coefficient of subgrade reaction was chosen as  $7 \text{ MN/m}^3$ increasing linearly with the depth and limited to 49MN/m<sup>3</sup> for all 2D cases (Trafikverket, 2011b).

Table 1 Studied cases.				
weight	<b>(kg/m</b> <sup>3</sup> )	500	1200	2000
Friction angle	(deg)			
25		Х	Х	Х
35		Х	Х	Unable to finish
45		Х	Х	Х

# 6 RESULTS

### 6.1 Validation of the numerical model

The numerical model was initially validated against the measurements presented in Takahashi, 1985. The maximum bending moment at each settlement level was calculated, following the distribution shown in Figure 3. The maximum bending moments typically occurred close to the surface at the same vertical level, showing relatively small change during the soil settlement process, (Takahashi, 1985). Figure 6 shows the bending moment against the ground surface settlement in the numerical model and the field measurements. It can be observed that the results are very close to the measured values and it is assumed that the model replicate the field test relatively well. From the results it can also be observed that a wedge like failure mode occurs as shown in Figure 7 indicating that a different failure mechanism occurs in the first few meters than in the rest of the pile.



Figure 6 Maximum bending moment plotted against the settlement of ground surface.



Figure 7 Plastic zones in the completed finite element solution indicating a different failure mode in the top of the pile.

As the 3D finite element model is assumed to replicate the results from the field tests, the validated numerical 3D-FEM model was then adapted for comparison to the different 2D discretization approaches with the field measurements in Takahashi, 1985.

# 6.2 Beam-spring model according to Svahn & Alén, 2006

Figure 8 shows the bending moment calculated according to Svahn and Alén, 2006, compared to the simulation by the 3D-FEM model. It can be seen that the proposed model gives a maximum bending moment far greater than that obtained in the 3D model.



Figure 8 Maximum bending moment plotted against the settlement of ground surface using *PKR101*.

6.3 Results using the distributed load approach according to Takahashi, 1985

Figure 9 shows the bending moment calculated according to the distributed loading approach, (Takahashi, 1985), compared to the ground surface settlement calculated with the 3D-FEM model. It can be seen that the proposed model gives a maximum bending moment close to the one obtained in the 3D model.



Figure 9 Maximum bending moment plotted against the settlement of ground surface using the distributed load approach.

# 6.4 Results using the wedge failure approach according to Reese, 1974

Figure 10 shows the bending moment calculated with the wedge approach according to Reese et al, compared to the ground surface settlement calculated with the 3D-FEM model. It can be seen that this model gives a maximum bending moment close to the one obtained in the 3D model.



Figure 10 Maximum bending moment plotted against the settlement of ground surface using the wedge failure approach.

## 6.5 Results from the parametric study

After assessing the different between the beam-spring approaches compared to the field measurement in Takahashi, 1985, a parametric study was carried out. The soil friction angle and the weight of the soil were varied according to Table 1. The results are shown in Figure 11. The abscissa shows the effective weight of the soil and the ordinate the friction angle. It was first be observed that the proposed beam-spring models (Svahn and Alén, 2006, Takahashi, 1985 and Reese et al, 1974) results in different bending moments depending on the friction angle and the weight of the soil. Moreover, it is also noticeable that the distributed load model is independent of the friction angle but gives a maximum bending moment closer to the one obtained in the 3D models in comparison to the wedge failure method. All three ways of calculating the reaction of the pile give results on the safe side, however by using Svahn and Alén, 2006, the calculations results in a bending moment of over 3 times the value obtained from the 3D-FEM simulation.



Figure 11 Maximum bending moment in pile divided with the maximum bending moment for a friction angle of 35 degrees and a weight of 1.2  $t/m^3$  plotted against the weight of the soil.

### 7 DISCUSSION

Ground settlement, including soil creep, occur over extended time periods, and many factors such as ground water level and presence of organic soil influence the settlement profile and settlement rate. A relatively simplified soil deformation model has been adapted to the inclined pile in settling to simulate the drained soil conditions in the current paper. The vertical strain profile was imposed on the soil according to the field test case, and a Mohr-Coulomb yield model was included in the model to simulate the plastic behaviour during settlement. The results are in line with the wedge yield theory discussed in literature, e.g. Randolph 2014, Reese 1974, Takahashi 1985. It appears from the results that the main response of the soil can be separated into a surface field mechanism, and a deeper earth pressure mechanism, following the standard theory of beam-spring models for horizontal offshore piles, (Randolph, 2014). A numerical parametric study of the variation of the soil weight and friction angle confirms the importance of the geometry of the soil on the inclined pile, in which the bending moments did not change very much when these factors were varied. The mechanism with the least necessary resistance before yield controls the position of transformation between the top and deep yield type through the wedge mechanism in the soil. Because of the relatively large displacement in the top soil layer at yield, more advanced soil models incorporating small-strain behaviour would probably have limited impact on the simulation. However, a soil model including viscoelastic behaviour such as creep would probably improve the numerical model.

#### 8 CONCLUSIONS

Analysis of imposed vertical deformation (simulating ground settlement) shows that a wedge-type yield mechanism occurs in the top part of the soil. This mechanism is not correctly simulated by the current calculation model (Svahn and Alén, 2006), in which an earth-pressure formulation is adapted along the whole pile depth. The alternative beamspring models that differentiate between the top layer and bottom layer with either a wedge, e.g. (Reese et al), or with a distributed load depending on the width of the pile, results in a more realistic idealization of the real case. Numerical simulations with a beam-spring model formulation were carried out with the wedge failure (Reese et al, 1974) and the distributed load approach (Takahashi, 1985) and compared with the 3D-FEM model. A subsequent parametric study was also carried out. The simulation results show that the distributed load approach results in calculations which are relatively similar to the 3D-FEM model. The beam-spring distributed load approach (Takahashi, 1985) with a drained earth-pressure formulation is therefore proposed as the preferred design approach for inclined piles in settling soil, both for clay and granular soils. Another conclusion of the numerical simulations is that beam-spring family of models are very simplified design models, consisting of simplifications and idealization of relatively multi-faceted mechanical response of the soil around the pile. Such models should be adapted to design with some care, preferably after a full simulation with a more realistic model. The stress distribution around the pile in the case of settling soil is quite different from that of direct horizontal pile loading, in which case another of the alternative formulations are preferred for granular soil and clay, (Randolph, 2014). The settlement of clay soil is a very slow process that is likely to give a drained soil response, while much faster loading, e.g. traffic load, results in excess pore pressure, and the presented model is not suitable in such a case.

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